AD NUMBER
AD045696
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JUN 1954. Other requests shall be referred to Wright Air Development Center, Wright-Patterson AFB, OH 45433.**
AUTHORITY
AFAL ltr, 27 Dec 1979

UNCLASSIFIED

AD 45 696

Reproduced
by the
Armed Services Technical Information Agency

ARLINGTON HALL STATION; ARLINGTON 12 VIRGINIA

UNCLASSIFIED

WADC TECHNICAL REPORT 54-272

ESTABLISHING VIBRATION AND SHOCK TESTS FOR AIRBORNE ELECTRONIC EQUIPMENT

CHARLES E. CREDE
MAURICE GERTEL
RICHARD D. CAVANAUGH

THE BARRY CORPORATION

JUNE 1954

WRIGHT AIR DEVELOPMENT CENTER

NOTICE

WADC TECHNICAL REPORT 54-272

ESTABLISHING VIBRATION AND SHOCK TESTS FOR AIRBORNE ELECTRONIC EQUIPMENT

Charles E. Crede
Maurice Gertel
Richard D. Cavanaugh

The Barry Corporation

June 1954

Electronic Components Laboratory
Contract No. AF 33(038)-22704
Task No. 41610

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

Carpenter Litho & Prtg. Co., Springfield, O.
100 - 26 October 1954

FOREWORD

This report was prepared by Charles E. Crede, Maurice Gertel, and Richard D. Cavanaugh of The Barry Corporation, Watertown, Massachusetts, and summarizes the results obtained during October 1951 to June 1954 on Air Force Contract No. AF 33(038)-22704, under Project No. 4157, Task No. 41610 (formerly R-112-143-A) "Vibration and Shock Design Criteria for Electronic Equipment." The work was administered under the direction of the Electronic Components Laboratory, Wright Air Development Center, with Mr. C. A. Golueke initially in charge of the project. He was later succeeded by Mr. N. P. Kempton as project engineer.

ABSTRACT

The results of an analysis of the vibration and shock environment in piloted aircraft are presented. The objectives of the study were to define such environment and to devise appropriate laboratory tests for certifying the suitability of electronic and accessory equipment for use in aircraft service. The analysis used in the study is based on the thesis that the vibration and shock as measured in aircraft are relatively unimportant, but that the response of equipment to such vibration and shock is of paramount importance. This follows from the fact that the stress experienced by structures of the equipment is directly related to the response of such structures to the applied vibration and shock.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

for RICHARD S. CARTER
Colonel, USAF
Chief, Electronic Components Laboratory
Directorate of Research

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS - CONTINUED

APPENDIX I

APPENDIX II

SYMBOLS

b = a coefficient, integer or fraction

D = number of landings in life of equipment

D' = number of applications of shock in shock test.

f = forcing frequency, cps.

f_n = natural frequency, cps.

h = rate of change of test frequency, cycles/sec./sec.

k = stiffness of equipment element, lb./in

m = mass of equipment element, lb.sec.^{2}/in.

n = number of cycles of stress of a designated magnitude for one application of environmental shock

n' = number of cycles of stress of a designated magnitude for one application of laboratory shock

n_t = total number of cycles = Dn

n_t' = total number of cycles = $D'n'$

N = cycles to failure in fatigue test

Q = value of transmissibility at resonance in steady-state vibration

r = number of cycles of free vibration

R = a particular value of r

R' = a constant

S = maximum stress in fatigue test, psi.

x = vibration amplitude of airframe structure, half peak-to-peak, in.

x_o = maximum value of x, in.

$\ddot{x}_o$ = maximum acceleration associated with x, g

y = vibration amplitude of equipment element, half peak-to-peak, in.

y_o = maximum value of y

$\ddot{Y}_0$ = maximum acceleration associated with Y, g

ω = forcing frequency, rad./sec.

Ω = natural frequency, rad./sec.

INTRODUCTION

Equipment mounted in aircraft is subjected to vibration and shock of various degrees. It is important to the designers of such equipment that techniques be available for proving the suitability of the equipment for use in these shock and vibration environments. In many fields not related to aircraft, such answers are obtained by a direct approach. Automobile designs are proved, for example, by operating the automobile under actual but severe conditions for a period corresponding to its maximum expected life. Washing machines are tested in a somewhat similar manner in the laboratory, and internal combustion engines are proved by operating them at maximum load for a period corresponding to the life of the engine. These techniques are not feasible for electronic and accessory equipment installed in aircraft for reasons which may be summarized as follows:

(a) Equipment of a given design usually is installed in many different types of aircraft. Those aircraft encounter many different operating conditions, and it becomes impractical to test a given equipment in environments representing all possible operating conditions in all types of aircraft.

(b) Electronic and accessory equipment, in general, is not designed or manufactured by the airframe manufacturer. The equipment must be received ready for installation in the airframe with assurance that it will render satisfactory performance during all conditions of flight.

This report analyzes some of the problems introduced by the above circumstances, and suggests a rational procedure for establishment of laboratory tests. The objective of such tests is to qualify equipment for operation under actual aircraft operating conditions.

If the procedure referred to above for testing automobiles were to be used for qualifying electronic equipment intended for airborne service, it would be necessary to first measure all conditions of shock and vibration encountered in all aircraft in which the equipment is to be installed. The next step would be determined the desired life of the equipment in hours. One equipment would then be subjected to each measured condition of shock and vibration for a period equal to the desired life of the equipment. This would require that a new equipment be made available for each test condition, because damage from vibration and shock tends to be cumulative and the results would not be significant if some of the tests were conducted on equipment previously subjected to environmental tests.

It is well known that failure caused by repeated application of stress tends to be erratic. Consequently, many samples would have to be tested under identical conditions so that the results could be examined statistically. The quantity of equipment and the testing time required for such a program would be exorbitant.

One method of resolving the question of an unreasonably long testing procedure is to adopt a concept of accelerated testing. It is generally assumed that equipment tends to sustain the same damage as a result of mild vibration or shock for a long period of time as it would sustain as a result of severe vibration or shock for a short period of time. This philosophy appears to underlie many of the vibration and shock tests now required by military specifications. In most cases, the tests appear to have been established arbitrarily and without a rational study of the effect of increased severity. To the best knowledge of the authors, the concept of accelerated testing exists only qualitatively, and no critical analysis of the problem has been made to establish a quantitative basis of accelerated testing. An attempt is made in this report to fill this deficiency by examining critically and quantitatively the problem involved in establishing vibration and shock test specifications based on accelerated testing.

A detailed consideration of the subject of accelerated testing reveals many diverse and complex problems, some of which may be outlined as follows:

(a) A criterion of failure must be established. It is common experience, borne out by recent studies, (see reference 80 of Appendix III), that certain types of equipment tend to deteriorate gradually during use and in the absence of significant shock and vibration. Depending on its nature, this normal deterioration may or may not be accelerated in the same manner as damage resulting from shock and vibration. If the deterioration is not accelerated by shock and vibration tests, such tests tend to be unconservative because greater damage would occur if the tests were continued for a longer period.

(b) Questions of both operation and strength become involved in any consideration of accelerated testing. Equipment should be required to operate only at the maximum severity of vibration and shock expected to be encountered in actual operating conditions. The increased severity associated with accelerated tests should be applied only to investigations of the strenth of the equipment from a mechanical or structural standpoint. There should be no requirement that the equipment continue to operate properly under these severe conditions.

(c) Many types of failure occur in equipment subjected to shock and vibration. These are difficult to categorize in general. A review of damage reports indicates that failure of brackets and other structural members as a result of repeated stressing is common. Another group includes somewhat similar failure of wires, tube elements, and other electrical components. A third group includes loosening of fasteners, crumbling of ceramic and mica insulators, and similar failures that may or may not have any relation to the magnitude of the stress existing in the element.

IDEALIZATION OF EQUIPMENT

The problems involved in analyzing the various types of failures may be considered by creating a hypothetical equipment having components of the types known to involve failure. It may then be possible to study the manner in which the various parameters affect the likelihood of failure. The hypothetical equipment to be discussed is illustrated schematically in Figure 1. This equipment includes a housing having a horizontal panel on which several components are mounted. The components include vacuum tubes A and B, a condenser C secured to the panel by the bolts D, and a resistor E fastened by its electrical leads F to binding posts G mounted upon the panel. Failure of the equipment may occur as a result of various effects discussed in the following paragraphs.

If failure of the panel occurs, it is probable that the failure is the result of excessive stress in the panel. It may be assumed that the stress in the panel is proportional to the vibration amplitude of the panel relative to the housing. Probability of failure thus tends to increase as the vibration amplitude of the panel increases.

Vacuum tube A is of a type in which the internal elements are supported entirely from the base structure. It may be assumed that failure tends to occur when these elements deflect excessively with respect to the base. The maximum deflection of the elements is proportional to the maximum acceleration of the panel, multiplied by a dynamic factor which takes the frequency relation into account. Except for this frequency relation and others to be considered below, the damaging potential applied to tube A tends to be proportional to the vibration amplitude of the panel on which the tube is mounted.

Tube B is somewhat similar to tube A, but includes the additional feature of a mica spacer adjacent to the upper ends of the internal elements to maintain such elements in properly spaced relation to each other and to the envelope of the tube. The forces applied to the spacer are a function of the tendencies of the elements within the tube to deflect. Such tendencies are related to the vibration amplitude of the panel as discussed previously with reference to tube A. The damaging potential on the mica spacer is therefore related, but in a somewhat less definite way, to the vibration amplitude of the panel which supports the tube.

Components mounted to the panel by brackets, such as the condenser C tend to fail as a result of excessively high stress in the mounting brackets. The sole function of such brackets is to mount the condenser to the panel of the equipment, and the stress in the brackets tends to be directly proportional to the inertia forces involved in causing the condenser to move in the same nanner as the panel upon which it is mounted. The damaging potential on these brackets thus tends to be directly proportional to the maximum vibration amplitude of the panel.

Failures may occur as a result of loosening of bolts D. In spite of many investigations and considerable analysis, the laws governing the loosening of bolts are not well established. One possible explanation is that the bolt stretches somewhat under the influence of dynamic forces, with a consequent momentary release of the friction force between the face of the nut and the bolted surface. This momentary release of friction encourages the nut to back off the bolt. It may thus be assumed that the loosening tendency increases as the vibration amplitude of the panel increases.

The problem of the resistor E is somewhat similar to that of the condenser C expept that the mounting wires have the additional function of providing an electrical circuit to the resistor. These wires also constitute the mechanical support for the resistor. The maximum stress in the wire tends to increase as the vibration amplitude of the panel increases, for the reasons discussed with reference to the brackets which support the condenser C.

Although subject to many qualifications, it may be stated that, to a first approximation, the likelihood of damage to the hypothetical equipment illustrated in Figure 1 tends to increase as the vibration amplitude of the mounting panel increases. This simplified approach to a complicated problem thus suggests the designation of the vibration amplitude of this panel as the single parameter that may serve as an index of likelihood of failure of the electronic equipment when subjected to vibration and shock. Problems involved in determining this amplitude are considered below in great detail for the various kinds of motion embodied in vibration and shock environments.

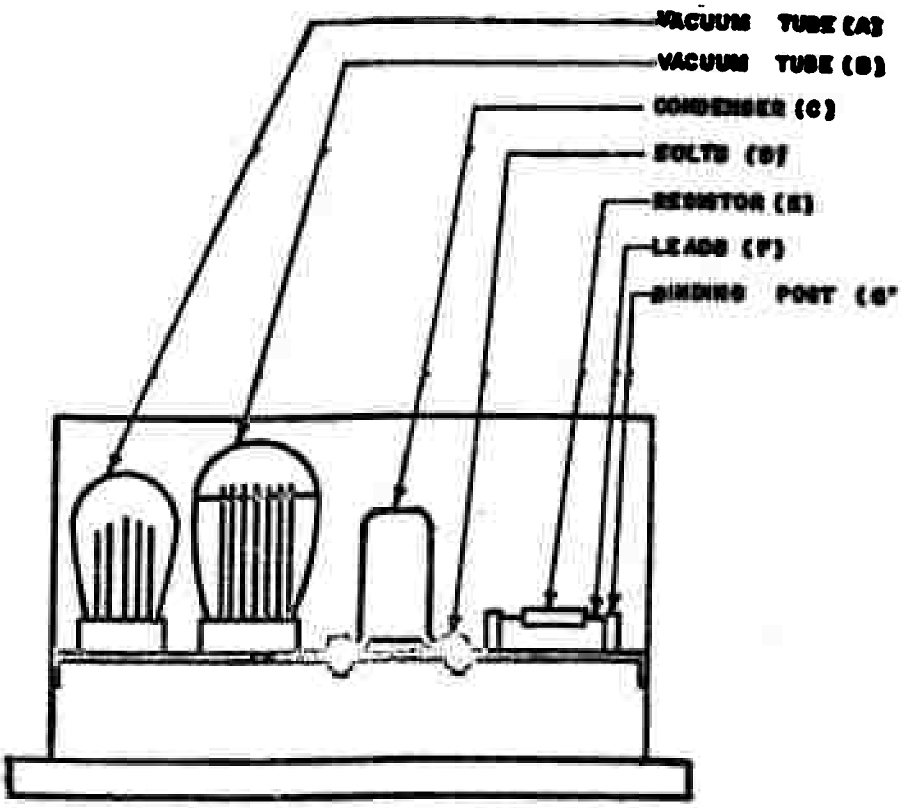


FIGURE 1. SCHEMATIC DRAWING OF HYPOTHETICAL EQUIPMENT HAVING TYPICAL COMPONENTS

VIBRATION ENVIRONMENT

The words "shock and vibration" are commonly used together to describe certain aspects of the environmental conditions which exist on aircraft. The use of the two discrete words implies a distinction between two classes of phenomena. The meaning of the word "vibration" is much better established in a technical or engineering sense. In general, it refers to a periodically varying force or motion which many be either steady-state or transient in nature. Vibration is defined here as a vibratory motion of the aircraft structure which is steady-state in nature and may consist of one or more frequencies with the motion at each frequency being harmonic. Steady-state vibration may be completely defined by designating the frequency or frequencies, together with the amplitude at each frequency. The amplitude is commonly defined in terms of displacement, velocity or acceleration.

The meaning of the word "shock" tends to be vague and indefinite. It carries with it a connotation of suddenness, and perhaps also of significant over-all motion. In impact of rigid bodies, the velocities of the bodies change instantaneously, and a condition of shock clearly exists. Shock in aircraft may, and often does, originate from suddenly applied forces. These forces result from gun fire, landing, aerodynamic buffeting, and similar conditions. The structures of aircraft are generally light and non-rigid, and therefore incapable of transmitting suddenly applied forces. The impact forces, instead, tend to excite transient vibration of the aircraft structure. This transient vibration often is of substantial amplitude and occurs at many frequencies which are generally dictated by (1) the natural frequencies of the aircraft structures and (2) the frequency components in the applied impact force. In this report, an oscillating motion will be referred to as vibration if it is sufficiently regular that it can be defined by a frequency or frequencies together with the steady-state amplitude at each frequency. If it does not meet these requirements, it must be considered transient in nature, and will be referred to as a shock motion. It is not possible to define a shock motion by assigning numerical values to established parameters. A shock motion can be defined adequately only by describing the time history of a physically significant parameter, such as acceleration, velocity or displacement.

Although shock or transient vibration can be described only by an oscillogram setting forth the time history of acceleration, displacement or velocity, such oscillograms are not included in reference 55 of Appendix III which nominally is the basis for the present study. It has been necessary to go beyond the initially intended scope of this investigation and to obtain oscillograms of vibration and shock conditions suitable for analysis leading toward the establishment

of laboratory testing procedures. A number of oscillograms giving acceleration as a function of time have been made available to the Contractor by the Aircraft Laboratory, Wright Air Development Center, Wright-Patterson Air Force Base. From among this group of records, selections were made for analysis. All of the records analyzed were the results of measurements made at the center-of-gravity of the respective aircraft during a landing. Insofar as could be determined prior to analysis, the records were selected on the criterion that they be the most severe of the group, and represent a diversity of characteristics so that the results would be representative of a range of shock conditions. The selected oscillograms are set forth as insets to the several sheets comprising Appendix II, wherein the particular aircraft are identified. The details of the analysis carried out on these records are discussed in a subsequent section of this report.

It is evident that the vibration environment in aircraft during actual operating conditions must be known to form a basis for the establishment of laboratory testing procedures. Vibration in aircraft has been measured by many agencies. These agencies used a wide variety of instrumentation and adopted various methods of reporting the data. All such data available several years ago were examined and correlated during a project initiated by Curtiss-Wright Corporation and completed by North American Aviation, Inc. The results of this analysis are set forth in reference 55 of Appendix III, a voluminous report containing a great mass of data. The usefulness of these data is subject to many qualifications as follows:

(a) The data are reported without regard to probable occurrence in actual service conditions. In the reference, the repeated recurrence of similar data may indicate only that trouble was being encountered under certain conditions, and that repeated measurements became necessary. Other conditions more likely to occur in service may be represented in the reference by relatively few data if there were no circumstances requiring repeated measurements. For these reasons, large concentrations of data, as reported in the reference, should not be accepted as evidence that the conditions described thereby predominate in actual service.

(b) Some of the data included in the reference represent unrealistic operating conditions, such as stalls and maneuvers that occurred during flights made to investigate air-worthiness of the aircraft. It is questionable whether such data should be included in the present study. If it is included, it is desirable to accord such data less significance than similar data recorded under standard operating conditions.

(c) Some of the data were taken on experimental planes, as indicated by the prefix X on the model number of the plane. In general, not more than a few such planes exist, and the conditions measured thereon are not necessarily typical of those that may be expected to be encountered by aircraft equipment in general.

(d) The reference includes a number of measurements made at locations on the airframe not suitable for mounting electronic and other accessory equipment. Inasmuch as the purpose of this study is to establish testing procedures for such equipment, it would appear evident that measurements made at such locations should be deleted from consideration in this analysis.

(e) The reference describes all vibration and shock environments in terms of numerically defined amplitudes and frequencies. There is no apparent distinction between transient and steady-state conditions. It thus becomes necessary to apply careful discrimination to the interpretation of the data to insure, if possible, that transient data are not being treated as steady-state data.

For the several reasons set forth above, it has been considered necessary to reconsider the environmental data set forth in reference 55 of Appendix III. These data are expressed in terms of double amplitude as a function of frequency. By screening the tabulated data in reference 55, the plot shown in Figure 2 was obtained. This figure includes data taken only from measurements on primary airframe structures of fuselage nose and center sections. Data obtained from measurements on equipments, experimental type planes, and obsolete planes have been excluded. An effort was made in the plotting of Figure 2 to include only the substantially steady-state vibration resulting from normal tactical flight. It is hoped that this figure excludes, to a substantial extent, data representing transient resulting from landing, gunfire, and unusual flight conditions. This cannot be guaranteed, however, because in many cases the actual flight conditions are not known.

An envelope indicated by line A in Figure 2 has been drawn around the spectrum of experimentally obtained vibration data. These data represent measurements made only on aircraft powered by reciprocating engines. Reference 55 includes relatively few measurements on airframe structures of jet-powered aircraft during conditions of normal tactical flight which would qualify for presentation in Figure 2. These data have been examined with regard to frequency and amplitude ranges, and insofar as it is

possible to judge, they fall within the envelope indicated by line A in Figure 2. The envelope A in Figure 2 may thus be considered representative of the envelope of maximum vibration conditions in all types of aircraft.

The data reported in reference 55 describing the environment in various types of aircraft powered by reciprocating engines have been examined statistically. The results of this statistical analysis are set forth in reference 70. This analysis reveals the trend of a median line which establishes the relation between frequency and amplitude. It also establishes a line parallel with the trend line below which 95% of all data lie. The inference to be drawn from the analysis is that the 95% line constitutes a reasonable expectancy limit for use in formulating specification requirements. It is set forth on Figure 2 as envelope B.

Existing data on steady-state vibration in Naval aircraft have been examined independently by personnel of the Bureau of Aeronautics of the Navy Department. As a result of this analysis, an envelope representing maximum severity of expected vibration has been formulated. This envelope is shown by line C in Figure 2. At the lower frequency part of the spectrum, the envelope has two branches. The lower of these branches is considered representative of the fuselage center section, while the upper is considered representative of the fuselage tail section and wing outer panels.

It is interesting to compare the envelope derived from a refinement of reference 55 with similar data complied in England. The latter are available in Specification G.100 of the Royal Aircraft Establishment. Data are given in this specification for several regions of aircraft. An envelope for the central region is indicated by line D in Figure 2, and a corresponding envelope for outer region of aircraft is given by line E in Figure 2. Central regions are defined as the main fuselage and inboard wing sections, while outer regions are described as constituting the tail and outboard wing sections. A comparison of the British data with that obtained from a refinement of reference 55 shows that the vibration amplitudes embodied in the British data are lower at low frequency vibration and higher at high frequency vibration. This leads to the suggestion that some of the low frequency data set forth in reference 55 may represent transient conditions. If this is true, such data should be deleted from the summary of steady-state conditions. It is unfortunate that the original data from which reference 55 was prepared are not readily available for review.

The vibration data presented in reference 55 refer predominantly to aircraft powered by reciprocating engines, although some data are available for aircraft powered by jet engines. The relative vibration levels in these two types of aircraft have led to much discussion, and many opinions have been expressed regarding the magnitude of the vibration existing in jet powered aircraft. The opinion of

British authorities on this subject, as expressed in Specification G.100, are of interest: "With the introduction of jet turbine and jet turbine propeller power units, early subjective opinions led to the belief that some relaxation of vibration test requirements would be possible, particularly at the lower end of the frequency range. Subsequent vibration measurements, however, have shown that although there may be considerable reduction in vibration level at moderate cruising speeds in still air, this is not the case during the more severe operational conditions such as high speed flight and maneuvers. In the more severe conditions, the amplitudes have reached, and in a few cases exceeded, the levels now quoted. Such high amplitudes have not been confined to any particular part of the frequency range, and no relaxation of the test amplitude frequency curves has therefore been made for jet powered aircraft from the evidence at present available."

The assignment of numerical values to the envelope of maximum vibration conditions is important. This envelope defines a region whose coordinates are amplitude and frequency, and in which all steady-state aircraft vibration is likely to occur. The significance of the envelope is that airborne equipment must be capable of withstanding for an indefinite period any combination of amplitude and frequency represented by a point on or below the envelope. Difficulties of interpretation arise because most of the data are the abstracted data set forth in reference 55, and the original data from which such abstract was drawn are not readily available. Several of the envelopes devised by other agencies and included in Figure 2 are subjected to the same limitations because they are based upon the same abstracted data. Taking such data at its face value, envelope A is indicated. Based upon limited information on the nature of available records, the authors of this report have some reservations regarding the validity of the envelope. These reservations are based upon the suspicion that certain of the environmental data have been recorded as steady-state, whereas they are in fact transient. It is hoped to have the opportunity to analyze certain basic flight data at a future time and thereby either confirm the established envelope or modify it to agree with measured environmental conditions.

Considering the vibration data reported in reference 55, the qualifications that apply thereto, and the interpretations of those and other data that have been made independently by other agencies, a composite envelope of maximum expected vibration has been drawn as follows:

A. displacement amplitude of 0.030 inch peak-to-peak at all frequencies between 10 and 50 cycles per second, and an acceleration amplitude of 4g at all frequencies between 50 and 500 cycles per second.

This composite envelope is indicated by line F in Figure 2. It is evident that this envelope is a good fit to test data and that it is consistent with other interpretations of the appropriate vibration envelope. Any apparent deviation of envelope F from experimental data is easily justified by the fact that the envelope may be expressed simply as a maximum displacement amplitude at low frequencies and a maximum acceleration amplitude at high frequencies. The envelope extends to approximately 500 cps, although few experimental points lie above 200 cps. This lack of data at high frequency may be the result of limited response of instruments or to deletions in the interpretation of the data. The envelope F in Figure 2 is redrawn in Figure 3 in terms of maximum acceleration as a function of frequency.

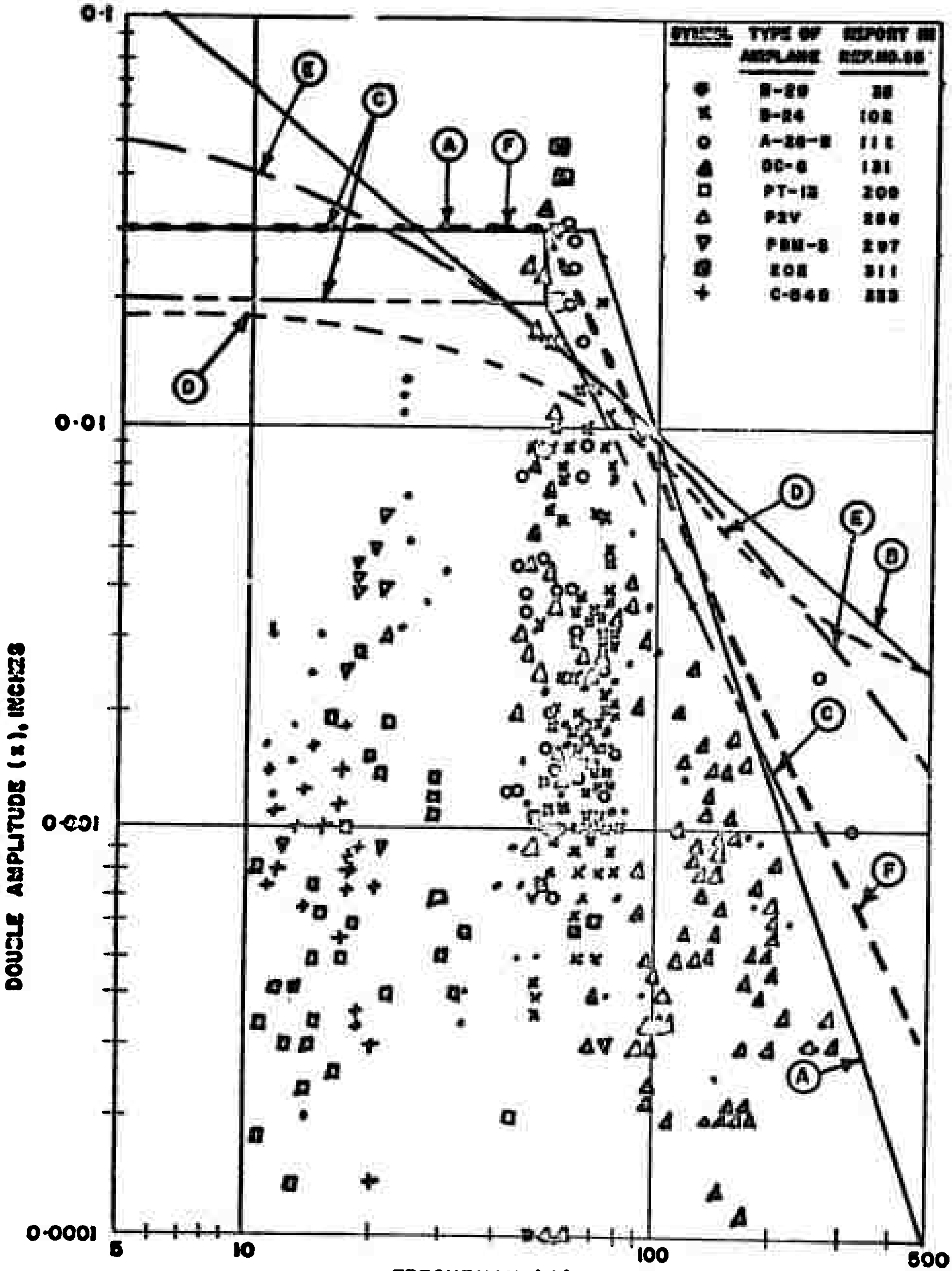


FIGURE 2· STEADY-STATE VIBRATION DURING NORMAL FLIGHT.

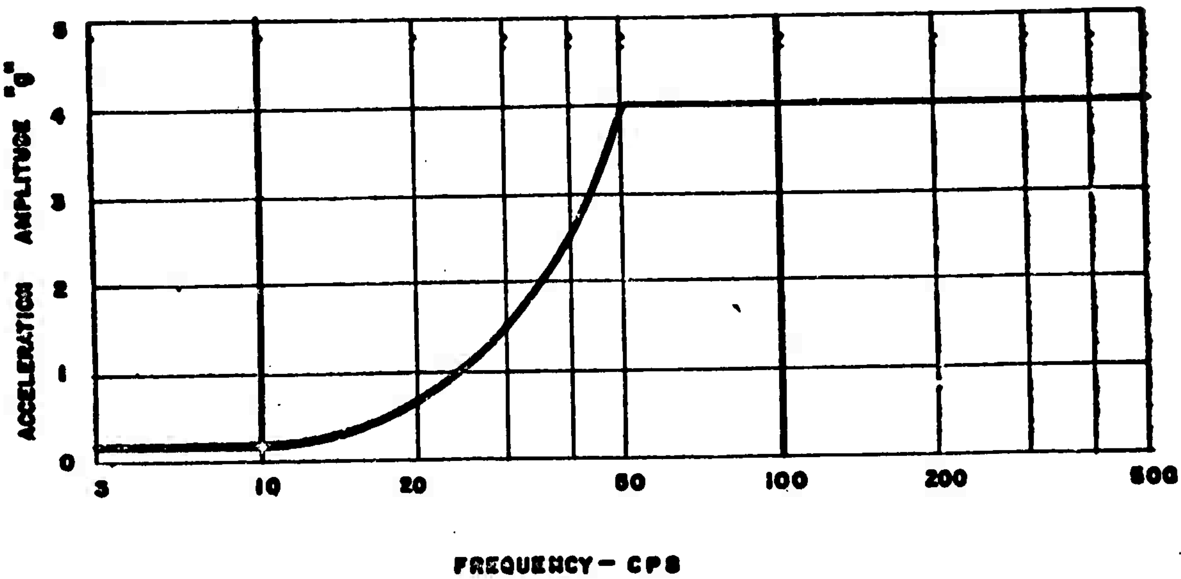


FIGURE 3. STEADY-STATE ENVIRONMENT ENVELOPE 'F' OF FIGURE 2 EXPRESSED IN TERMS OF ACCELERATION AMPLITUDE.

RESPONSE OF ELASTIC SYSTEMS TO STEADY-STATE VIBRATION

A vibration test embodying an exaggerated test amplitude intended to cause failure of the equipment after a relatively short time duration will be referred to here as an accelerated vibration test. An hypothesis of accelerated testing may be formulated by referring to the single-degree-of-freedom system shown in Figure 4. This system consists of a massless, linear spring k supporting a rigid mass m constrained by vertical guides to move only along a vertical line. The motion y of the supported mass is induced by the steady, simple harmonic motion x of the base. The motion of the base may be described by the following expression:

$$x = x_o \sin \omega t \tag{1}$$

where x is the displacement amplitude of the vibration of the base in the vertical direction and ω is the vibration frequency expressed in radians per second.

A complete solution of the differential equation describing the motion of this system includes terms representing the transient motion of the mass at the natural frequency of the mass-spring system. (See reference 2 of Appendix III). The natural frequency is expressed in units of radians per second by

$$\Omega = \sqrt{K/m}$$

and in units of cycles per second by

$$f_n = \sqrt{K/m}\,/2\pi$$

In any practical system, the spring will embody some damping, and these transient vibrations will be damped out ultimately. The motion of the mass may then be assumed to be simple harmonic at the same frequency as the motion of the base, but with a different amplitude. The relation between the motion of the mounted mass and the motion of the base is illustrated in Figure 5. The vertical scale is the ratio of displacement amplitudes, or since both x and y occur at the same frequency, the vertical scale also represents the ratio of acceleration amplitudes. The horizontal scale is the ratio of the forcing frequency ω to the natural frequency Ω of the mass spring system. For an undamped system, the curve theoretically reaches infinity at a frequency ratio of unity. This is a condition of resonance. All practical systems have some damping, and the amplitude at resonance is a finite value, as shown in Figure 5.

If the spring in the system of Figure 4 is linear and if

the mass is infinitely rigid, the relation between the acceleration of the mass and the deflection of the spring is given by the following equation:

$$m\ddot{y} = k(x-y) \tag{2}$$

Inasmuch as the stress in the spring is proportional to the deflection (x-y) of the spring, the stress is also directly proportional to the acceleration of the mass. This relation may now be extended by analogy to the hypothetical equipment illustrated in Figure 1. The parameter x may be considered to represent the motion of the airframe while the parameter y represents the motion of the center of the panel. The maximum expected motion of the airframe is defined by the acceleration-frequency plot shown in Figure 3 while the ratio of the maximum acceleration of the panel to the maximum acceleration of the airframe is given by Figure 5. Consequently, the maximum expected acceleration of the panel is the product of the ratio given in Figure 5 and the maximum acceleration of the airframe given in Figure 3. Repeated stressing of the panel thus occurs because its central part is being loaded with an inertia force proportional to the maximum acceleration of the center of the panel. Furthermore, components mounted upon the panel are also subjected to repeated stressing because the forces applied thereto increase as the vibration of the panel increases. This involves consideration of the strength of structural members which are subjected to repeated stresses.

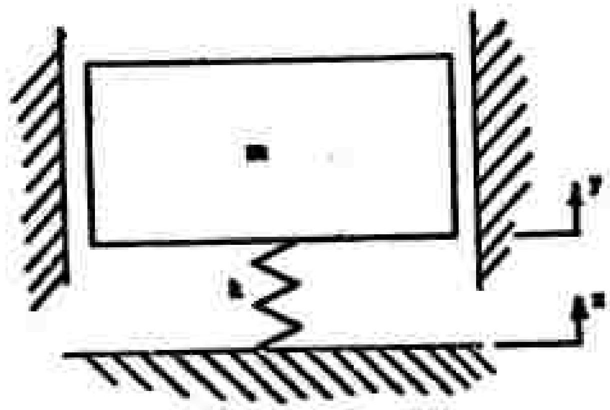


FIGURE 4. IDEALIZED SYSTEM REFERRED TO IN DEVELOPMENT OF HYPOTHESIS OF ACCELERATED VIBRATION TESTING.

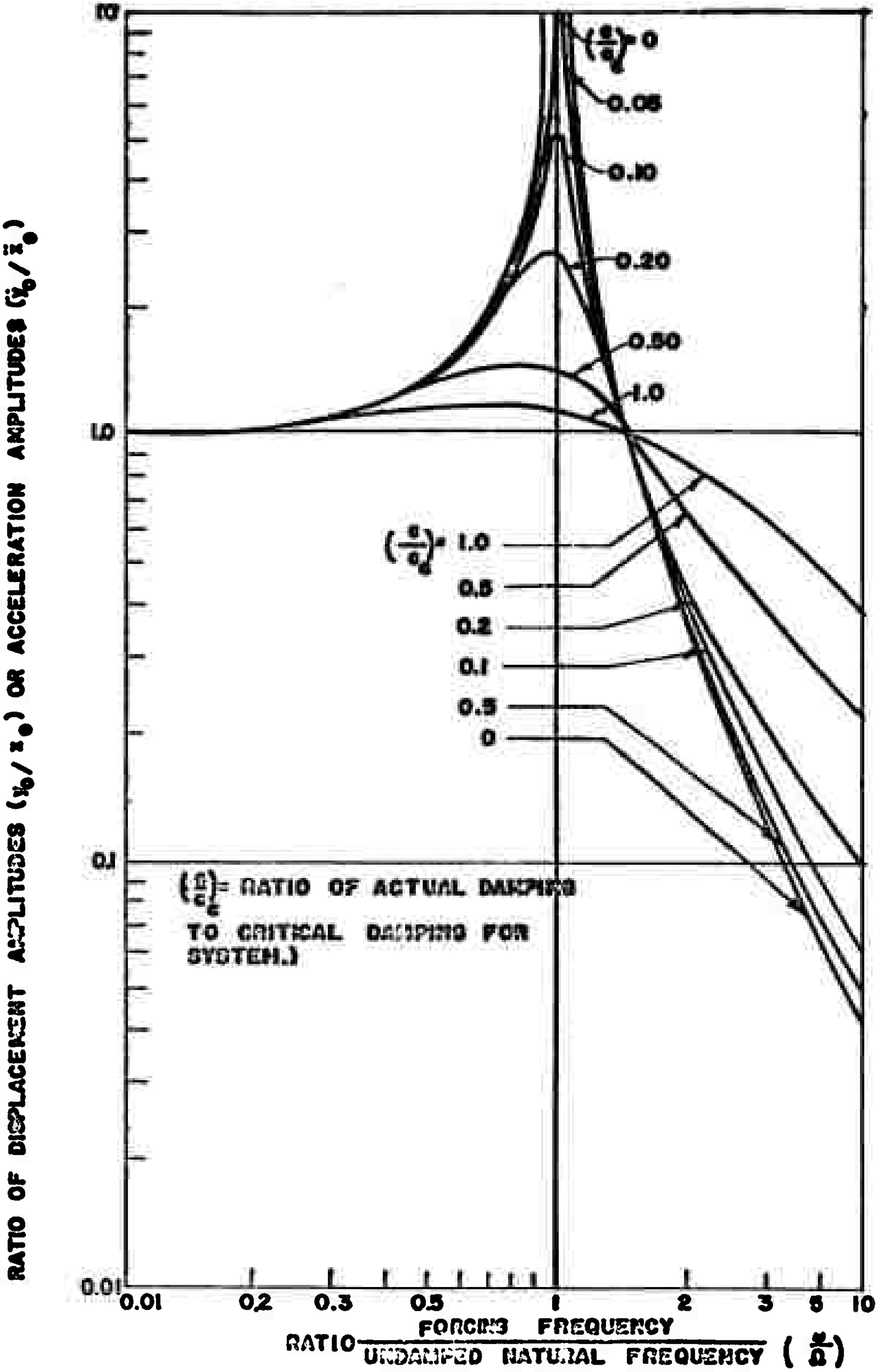


FIGURE 5. RATIO OF DISPLACEMENT AND ACCELERATION AMPLITUDES DESCRIBING MOTION OF SYSTEM IN FIGURE 4 IN STEADY-STATE VIBRATION.

CONCEPT OF THE FAILURE SURFACE

The properties of metals when subjected to repeated stressing have been well investigated, and much data exist in the technical literature describing the results of such investigations. The tests are usually conducted by manufacturing a number of test specimens which are as nearly identical to each other as possible. These specimens are used in testing machines that are arranged to impart alternating cycles of tensile and compressive stress to the specimen, or to impart bending stresses alternating in opposite directions. Tests are then run under similar conditions for each specimen, except that a different value of maximum stress is selected for each specimen in the test. Repeated applications of stress according to the selected stress pattern are then applied until the specimen fails. It is usually found that a specimen with a lower maximum stress will endure a greater number of cycles of stress reversal than a specimen with a higher maximum stress. The results are commonly reported as shown by the typical curve in Figure 6. This curve is commonly referred to as the $\underline{S}$-$\underline{N}$ curve, where $\underline{S}$ represents the maximum stress and $\underline{N}$ represents the number of cycles to failure. The exact shape of the curve differs somewhat from metal to metal. The curves for many metals have a well-pronounced knee, and the curve extends in a substantially horizontal direction rightwardly from the knee for an infinite number of cycles. This means that if the maximum stress is below the level represented by this horizontal part of the line, the specimen will endure an infinitely large number of cycles without experiencing failure. This stress level is referred to as the endurance limit for the material.

It was pointed out that the maximum stress in the spring $\underline{k}$ in Figure 4 tends to be proportional to the maximum acceleration $\underline{\ddot{y}_o}$ of the mounted mass $\underline{m}$. Consequently, the parameter of the ordinate scale in Figure 6 may be changed from maximum stress to maximum acceleration of the mass $\underline{m}$ if the endurance properties of the spring $\underline{k}$ are to be investigated. Since the maximum acceleration $\underline{\ddot{y}_o}$ of the mass $\underline{m}$ and the maximum acceleration $\underline{\ddot{x}_o}$ of the support are related as shown in Figure 5, the maximum acceleration $\underline{\ddot{y}_o}$ of the mass and the maximum acceleration $\underline{\ddot{x}_o}$ of the support are directly proportional for any given forcing frequency. This makes it possible to draw the family of $\underline{\ddot{X}}$ - $\underline{N}$ curves illustrated in Figure 7 by converting $\underline{S}$ in Figure 6 to the appropriate value of $\underline{\ddot{y}_o}$, and then multiplying by the applicable ratio obtained from Figure 5. The vertical scale $\underline{\ddot{X}_o}$ is the maximum acceleration of the support, the horizontal scale $\underline{N}$ is the number of cycles to failure, and each of the curves corresponds to a particular forcing frequency. Now just as each specimen has a characteristic $\underline{S}$-$\underline{N}$ curve as shown in Figure 6, each system may be considered to have a characteristic $\underline{\ddot{X}}$ - $\underline{N}$ - $\underline{f}$ relation as shown in Figure 7, where it is presumed

that the natural frequency of the system and the relation between S and $\ddot{y}_0$ are known.

The conditions defined by the family of curves in Figure 7 can be expressed more conveniently in three dimensions as the surface shown in Figure 8. This surface will be referred to as a failure surface. Each equipment has a failure surface that is typical of that equipment, and the surface fully defines the combination of parameters that will produce failure of the equipment. The parameter of the vertical coordinate axis if the maximum acceleration embodied in the vibration of the support in Figure 4, the left-hand axis is the test frequency in cycles per second, and the right-hand axis represents the number of cycles to failure.

The surface illustrated in Figure 8 is for a hypothetical equipment having resonant frequencies f_2 and f_4. Resonant frequencies are indicated by valleys in the surface, because a lower value of applied acceleration $\ddot{x}_0$ is required to cause failure if the test frequency coincides with one of the natural frequencies of the equipment. These valleys extend generally parallel to the $\ddot{x}_0$ - N plane so that any plane through the surface parallel to the $\ddot{x}_0$ - f plane shows depressions at the characteristic resonant frequencies. The failure surface for a single-degree-of-freedom system would show only a single valley, because such a system has only one natural frequency. The failure surface for a complex equipment would show many valleys, depending on the number of frequencies at which damaging resonances occur within the equipment. The valleys in a particular failure surface may be of different depths if the resonances that the valleys represent apply to structures with different degrees of damping.

Assuming that the failure surface is in existence for the equipment being considered, a specification for an accelerated test may now be formulated. A time in hours is first selected which represents the duration of the actual service in flight which the equipment is required to endure. A line a is then drawn on the f - N plane to represent the number of cycles at each frequency that would be required to reach the selected flight time. Theoretically, line a would be curved as shown in Figure 8 because the number of cycles in a given period increases with an increase in frequency. Practically, a plane may be used because the failure surface is parallel with the N axis for large values of N. A curved surface is then generated by a vertically extending line moving along the path a drawn on the f - N plane. The envelope $\ddot{x}_0$ - f representing the maximum severity of vibration conditions expected in flight, as set forth on Figure 3, is then transcribed onto the curved surface which was generated by moving a straight line along the path a in Figure 8. This envelope $\ddot{x}_0$ - f is designated by b in Figure 8.

If the intersection of the failure surface with the surface generated along line a falls below the envelope b of maximum vibration conditions, failure of the equipment may be expected during flight if the vibration environment in the particular aircraft is at the maximum expected level. The condition of predicted failure is indicated by the cross-hatched area in Figure 8. For this particular equipment, it is thus indicated that the component having a resonant frequency f_4 may fail before the equipment receives N_2 cycles of vibration. The probability of the equipment failing is equal to the probability that the vibration level in the particular airplane reaches the maximum expected level.

Inasmuch as Figure 8 predicts possible failure of the component with a natural frequency f_4 during flight conditions, the accelerated testing procedure for laboratory use should cause failure of the same component. The accelerated testing procedure is established by selecting a relatively small value for the number of cycles to failure, as designated by N_1 in Figure 8. In this instance, N_1 is made small enough to be feasible for laboratory testing. It is now necessary to construct a new surface whose coordinates are $\dot{x}$ and f, intersecting the N axis at the value N_1. This newly selected surface may be a plane as shown in Figure 8. If the selected test procedure applies, the same number of cycles of vibration at each test frequency; i.e., a shorter testing time at the higher frequencies, the surface $N = N_1$ will be a plane. If it is desired to conduct a test for equal periods at each testing frequency, the surface in the region of the number N_1 will be a curved surface generated by a vertical line following a curved path in the f - N plane. This corresponds to the procedure used to establish the surface representing flight conditions except that the number of cycles is small to correspond to laboratory conditions.

The laboratory test conditions, as represented by line c, are now inscribed upon the surface recently constructed in the region of the value N_1. If the laboratory test is to be valid, the line c defining the laboratory testing procedure must intersect the failure surface at the frequency f_4 but avoid intersection at the frequency f_2. This indicates that the component having a natural frequency f_4 will fail during laboratory testing. The validity of the laboratory test thus tends to be established, because it predicts the same type of failure predicted for service of the equipment in actual flight.

The hypothesis embodying the failure surface is difficult

to apply in formulating an accelerated testing procedure, primarily because the failure surface is not known to exist for any particular equipment. A tremendous amount of testing would be required to establish a single surface. If sufficient tests were conducted, it would undoubtedly be found that failure conditions would be described by a blanket of appreciable thickness rather than by a surface of zero thickness. This is expected because the scatter of results in endurance testing is relatively great, and scatter would tend to produce a blanket rather than a surface. In the hope that significant data exists from which a failure surface could be plotted, Contractor solicited test data from more than 300 potential sources of such information. The results obtained from these inquiries indicate that the data probably do not exist. It thus becomes necessary, in order to apply this hypothesis using a failure surface, to make certain assumptions as follows:

(a) It must be assumed first that the shape of the $\underline{\ddot{x}_o}$ - $\underline{N}$ curve is similar to the shape of the $\underline{S}$ - $\underline{N}$ curves obtained by subjecting specimens of material to repeated stresses under closely controlled conditions. This assumption may be valid for certain conditions in which the failure occurs as a result of repetition of excessive stress in the equipment being tested. In other circumstances, the failure may have little relation to stress, and the assumption of an analogy between the $\underline{\ddot{x}_o}$ - $\underline{N}$ and $\underline{S}$ - $\underline{N}$ curves may be unwarranted. Assuming the analogy to be valid, because no other assumption appears indicated, it may be assumed that the $\underline{\ddot{x}_o}$ - $\underline{N}$ curve is flat for values of $\underline{N}$ greater than five million, and has an established slope for values of $\underline{N}$ less than five million and greater than one thousand.

(b) It may be further assumed that all equipment installed in aircraft will be required to endure more than five million cycles of vibration. A scale of values may then be assigned to the $\underline{\ddot{x}_o}$ axis in such a way that the flat part of the $\underline{\ddot{x}_o}$ - $\underline{N}$ curve ($N > 5 \times 10^6$) is sufficiently far above the measured values of acceleration in aircraft service that the valleys of the failure surface do not fall below the environmental line (b in Figure 8) when large values of $\underline{N}$ are assumed.

(c) It may be assumed that the depth of each valley is proportional to the height of the surface in the region of the valley. This is equivalent to assuming that the damping associated with each resonant element is equal and does not vary with amplitude of vibration. It is with some misgiving that this assumption is made because damping is neither uniform nor linear in many instances.

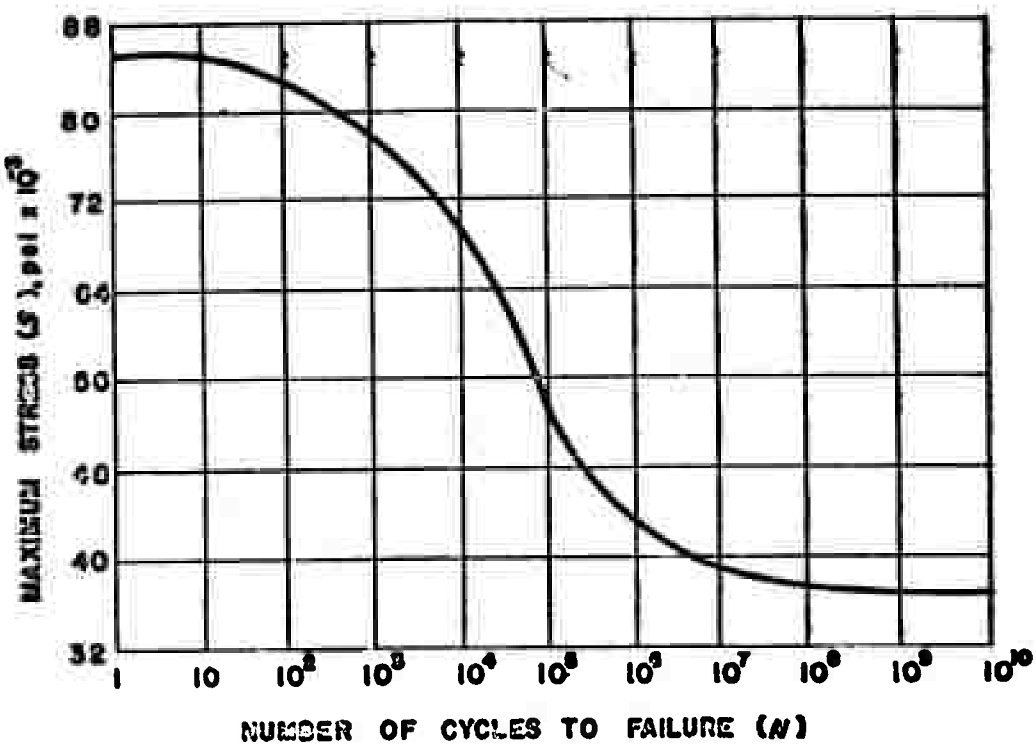


FIGURE 6. TYPICAL STRESS-CYCLE DIAGRAM SHOWING PROPERTIES OF STEEL WHEN SUBJECTED TO REPEATED STRESSING.

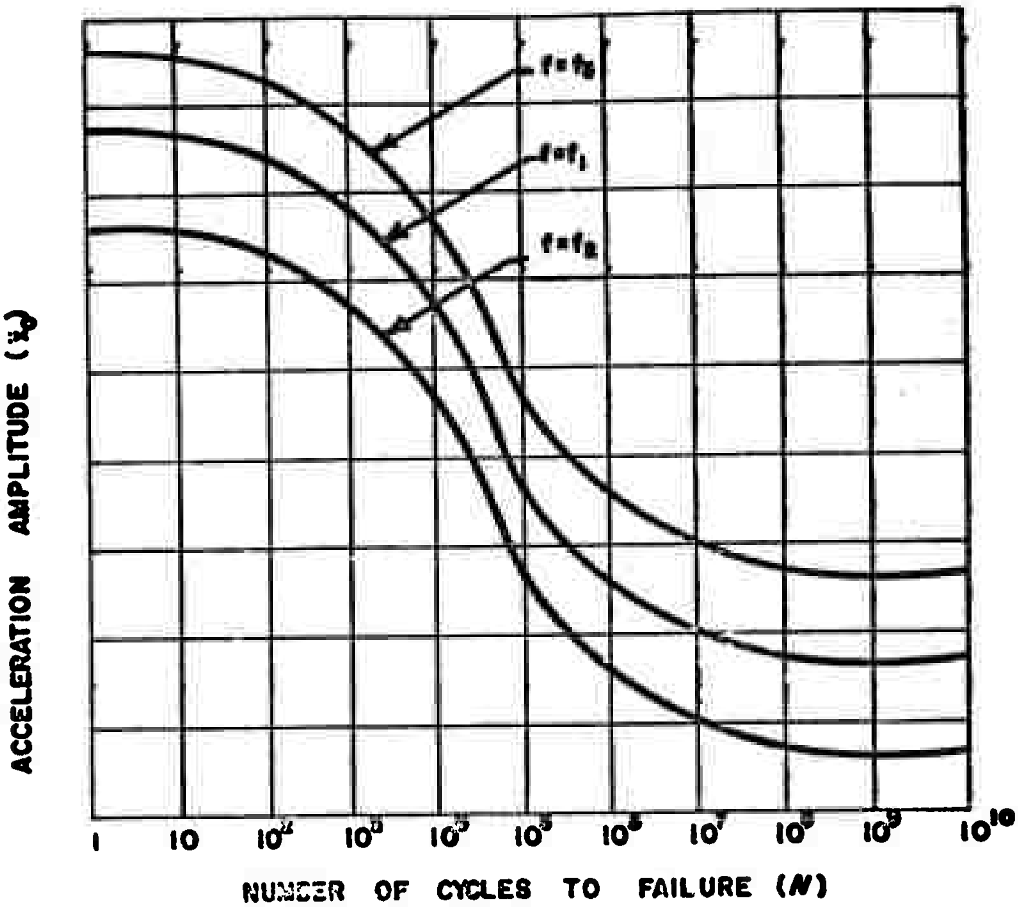


FIGURE 7· TYPICAL CURVE SHOWING ENDURANCE PROPERTIES OF SYSTEM ILLUSTRATED IN FIGURE 4, FOR A GIVEN NATURAL FREQUENCY AND KNOWN RELATION BETWEEN y_0 AND S.

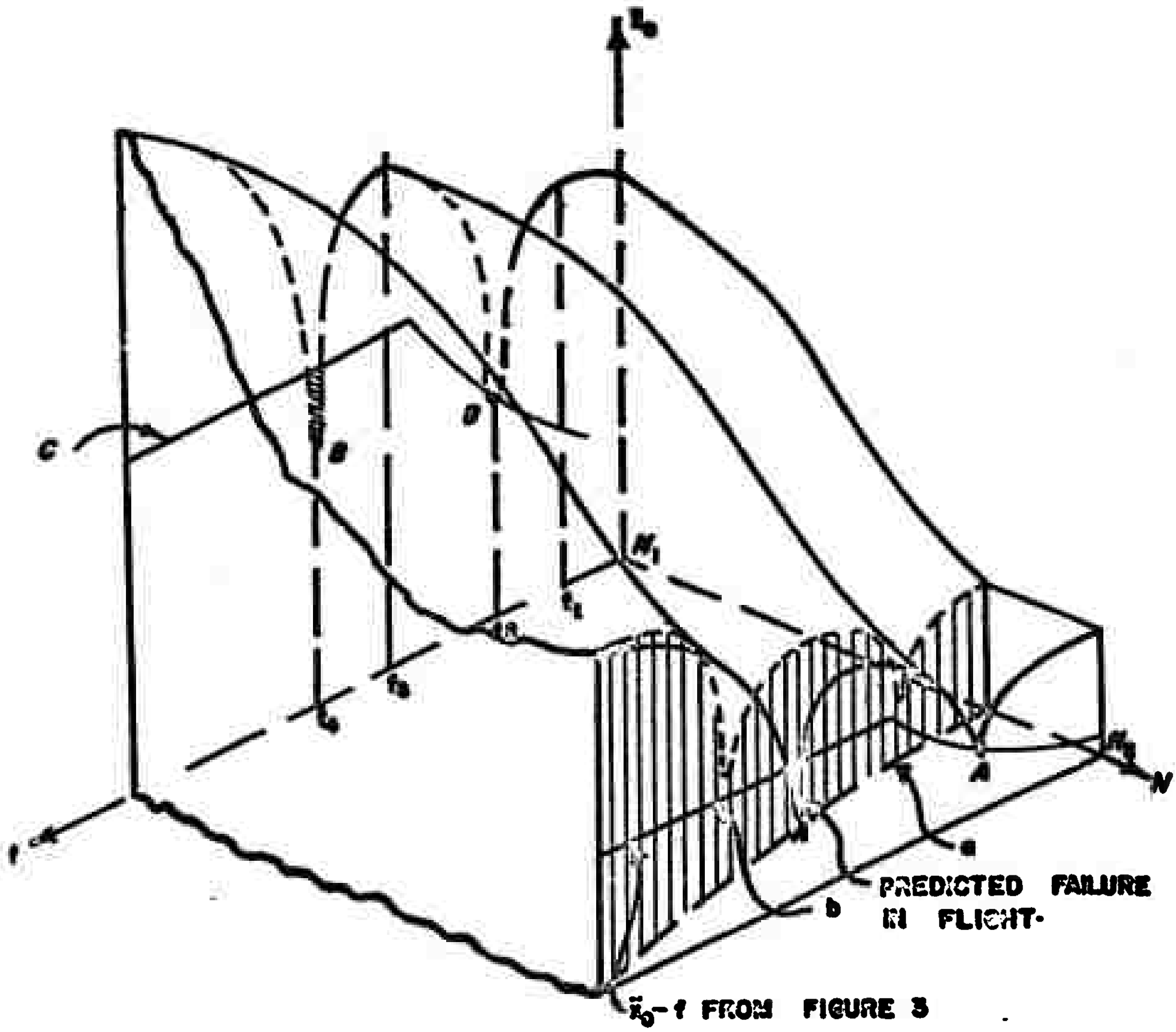


FIGURE 8. FAILURE SURFACE FOR EQUIPMENT HAVING COMPONENTS WITH NATURAL FREQUENCIES f_2 AND f_4, AND SUPERIMPOSED ENVELOPES SHOWING ENVIRONMENTAL AND LABORATORY CONDITIONS.

CORRELATION OF VIBRATION AND FATIGUE

The technical literature on fatigue or endurance testing has been very carefully surveyed in an attempt to establish the validity of assumption (a) in the previous section. The principal source of data has been reference 5. This survey has established the following numerical values which are assumed for purposes of this analysis to be representative of the properties of metals subjected to repeated stressing:

(a) The curve of maximum stress in steel as a function of number of cycles to failure is substantially flat for cycles greater than five million. In other words, if a test specimen made of steel has not failed after having been subjected to five million stress reversals, it is probable that failure will not occur if the test is continued indefinitely.

(b) The maximum stress at which infinite life of the test specimen is obtained is on a general average 38 per cent of the ultimate strength of the material. In other words, the maximum stress for which five million stress reversals may be obtained without failure is 38 per cent of the stress at which failure will occur upon one application of load. Further generalizing on the basis of the typical S-N curve shown in Figure 6, the maximum stress at 10^3 cycles to failure is assumed equal to the maximum stress at one cycle to failure.

The endurance or stress-cycle curve taken from the above generalization is shown by line I in Figure 9. Point A in Figure 9 corresponds with points A on the failure surface in Figure 8. The acceleration amplitude scales x_o are brought into correspondence by establishing a scale on Figure 9 so that the acceleration amplitude $\ddot{x}_o$ at point A on Figure 9 equals the acceleration amplitude x_o at the bottom of a depression in Figure 8 that is representative of the particular element considered vulnerable. This assumes that $N_2 > 5x10^6$ in Figure 8.

A question naturally arises concerning the validity of endurance tests of materials as a basis for establishing vibration test specifications. All known vibration test data bearing on this relationship have been studied. Useful data are extremely scarce, but the fragmentary information that is available tends to lend validity to the assumption:

(a) The Calidyne Company, under Contract #DA-36-039 SC-5545 with the U. S. Army Signal Corps Engineering Laboratories, conducted an investigation of fasteners for mounting electronic components. In the course of this work, extensive vibration tests were con-

ducted on complete equipments and on typical components. The components tested were not identical and it is, therefore, impossible to establish an endurance curve. The number of cycles of vibration necessary to cause failure of the various components has been analyzed, however, and the results are shown in Figure 10. It is evident that substantially all failures occurred at fewer than five million cycles of vibration, and that failure did not occur if the component withstood 3.4 million cycles. This tends to confirm the previous conclusion that a curve of acceleration amplitude vs. cycles to failure becomes horizontal for cycles greater than five million.

(b) Vibration tests to failure were conducted by Contractor, using a number of identical resistors. The resistors were supported by attaching the leads at opposite ends to spaced binding posts. Several resistors were grouped on a common panel and each group was subjected to vibration at a different level of maximum acceleration. Although the tests were not conducted at a resonant frequency of the resistors, the results are useful to establish the relation between acceleration amplitude and cycles of failure. Failure occurred generally by rupture of the electrical leads. The test results show considerable scatter, but the median is indicated by line II in Figure 9 in which the acceleration amplitude is adjusted to attain coincidence of lines I and II at $N=5x10^6$. A comparison of lines I and II in Figure 9 suggests agreement between the results of repeated stressing of metals and continued vibration tests of resistors. Insofar as the authors are aware, this is the best information available tending to establish a relation between vibration amplitude and number of cycles to failure.

Although the data discussed in the above paragraph tend to establish laws of mechanical failure, it is evident that electrical failure of the equipment is equally important. Data to establish the relation between severity of vibration and electrical failures are also scarce. Fragmentary data on failure of vacuum tubes as a result of vibration, based upon tests conducted by the Glenn L. Martin Company and by the Armour Research Foundation, have been obtained informally. Although the severity of these tests cannot be successfully correlated because resonance effects are not known, some indications of relative strength may be obtained. The test results are shown by line III in Figure 9, assuming the acceleration amplitude to be adjusted to obtain coincidence

with lines I and II at $\underline{N}$ = five million cycles. The drastically different slope of line III indicates a complete lack of agreement between electrical and mechanical failure. This tends to cast some doubt on the validity of the previous assumptions. The theory based upon mechanical failure should not be ruled out, however, until additional significant data on electrical failures are available. The existing data on electrical failures are presented here to illustrate the qualifications and limitations of the assumptions upon which this analysis is based.

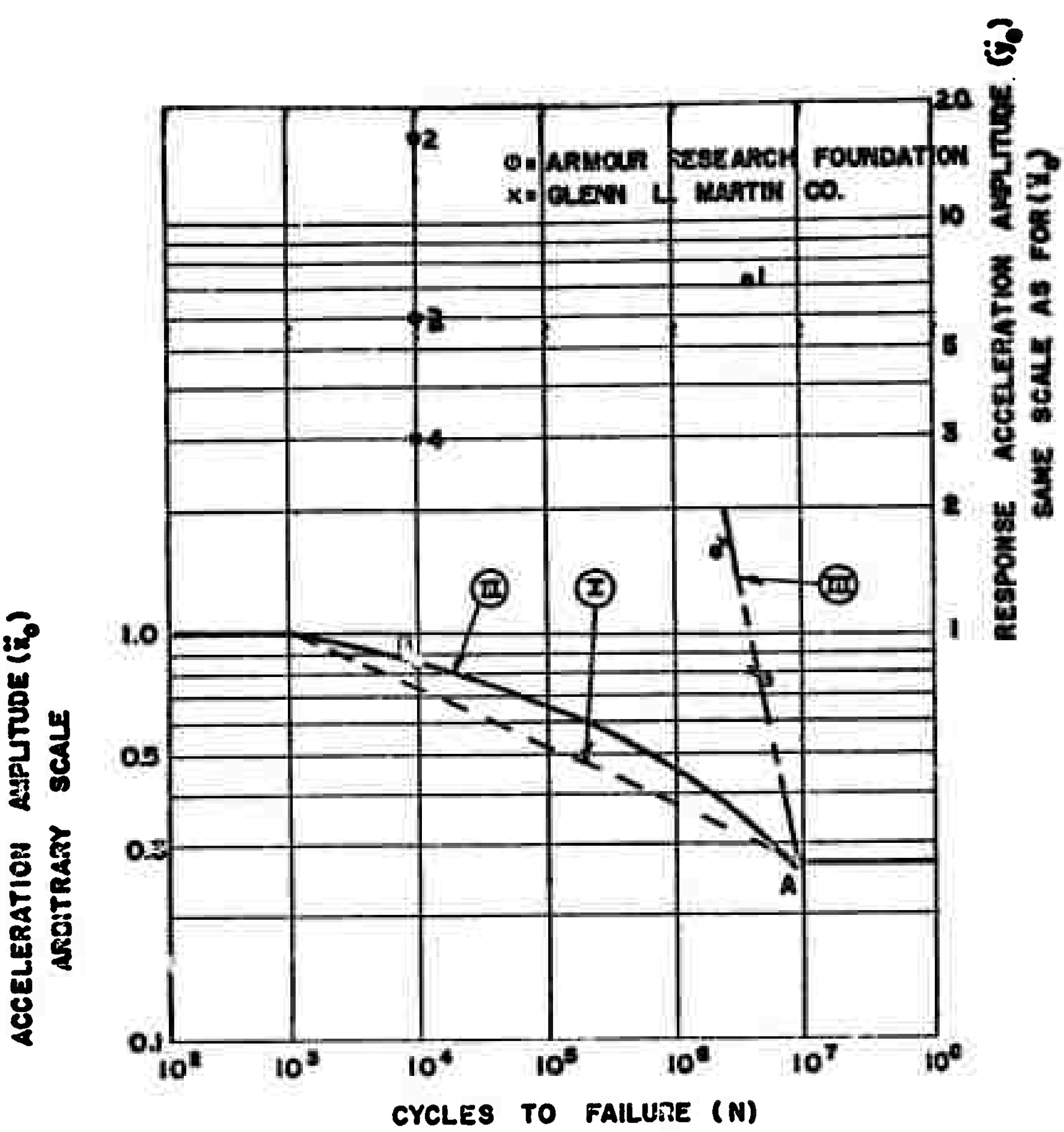


FIGURE 9. IDEALIZED ENDURANCE CURVE WITH SUPERIMPOSED DATA SHOWING RESULTS OF VIBRATION ENDURANCE TESTS.

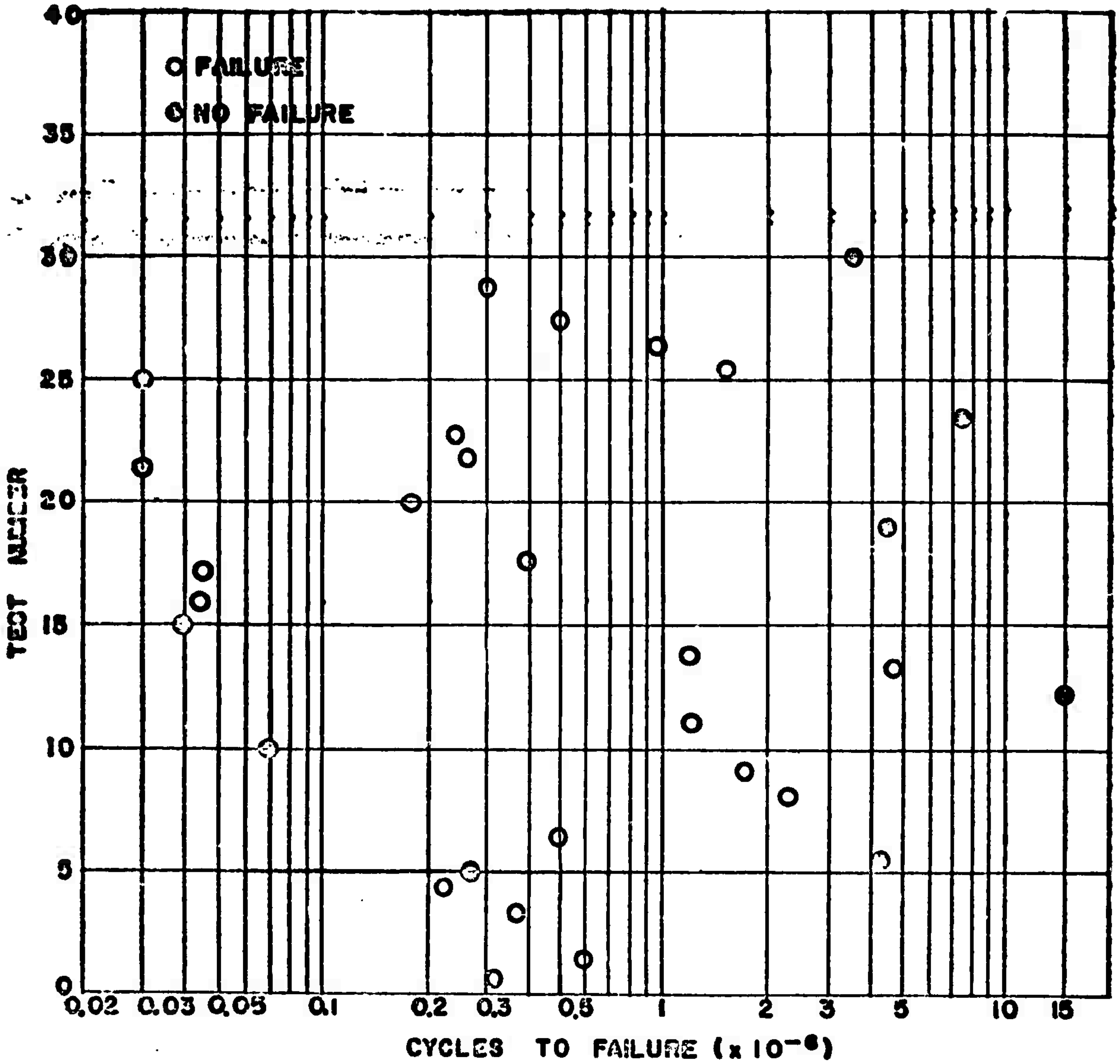


FIGURE 10. NUMBER OF CYCLES TO FAILURE IN VIBRATION TESTS CONDUCTED BY THE CALIDYNE CO.

CYCLING VS. DISCRETE TEST FREQUENCIES IN STEADY-STATE VIBRATION

If equipment fails during vibration tests, it is probable that failure occurs as a result of a resonant structure. The vulnerability of a structure in this respect is a function of the internal damping of the structure. If the damping is small, amplification at resonance is great and damage is more likely. Vibration tests are more difficult to conduct if the damping is small because the most damaging conditions may not be obtained unless the test frequency is carefully monitored. The following analysis is based upon an assumed internal damping equal to 2.5% of critical damping $c/c_c = 0.025$. This degree of damping gives an amplification at resonance of 20 by classical vibration theory. This is sometimes referred to as a system having a Q of 20. If the internal damping of the structure is greater than $2\frac{1}{2}\%$, the following analysis tends to be conservative as will become evident with reference to Figure 12. It is considered unlikely that the damping in any practical structure will be less then $2\frac{1}{2}\%$ for the relatively large displacement amplitudes being considered here.

It is generally conceded that the frequencies embodied in a vibration test should extend from the lowest to the highest found to be significant from a study of environmental conditions. One type of vibration test involves a continuous sweep of test frequency from the minimum to the maximum and back to the minimum. Another type of test involves vibration at a constant frequency for a predetermined period, followed by a period of vibration at a slightly different frequency, and followed by periods of vibration at other discrete frequencies until the entire range of frequencies is covered. The relative merits of these two types of tests will now be discussed.

As pointed out previously, Figure 9 represents a plane parallel to the plane of acceleration amplitude $\ddot{x}_o$ vs. cycles to failure N, taken through one of the valleys of the failure surface illustrated in Figure 8. This would be a critical test for a component having a natural frequency f_2 if the vibration environment were at the upper limit for such environment as established by the line b. Such a component would be expected to withstand five million cycles of vibration, as indicated by point A in Figure 9. Similarly, if $N_1 = 10^4$, it would be expected to barely withstand ten thousand cycles at a slightly higher value of acceleration amplitude, as indicated by point B in Figure 9. Points A and B in Figure 9 correspond to points A and B, respectively, in Figure 8.

In Figure 8, the values along the $\ddot{X}_o$ axis represent values of maximum acceleration embodied in the vibration environment. These correspond to acceleration associated

with the parameter x in Figure 4. These values of acceleration, when multiplied by the ratio $\ddot{y}_o/\ddot{x}_o$ set forth in Figure 5, give values of maximum acceleration $\ddot{y}_o$ associated with the motion of the mass m in Figure 4. As explained previously with reference to equation (2), the stress in the spring k is directly proportional to the acceleration $\ddot{y}$ of the mass m.

The relation between $\ddot{x}_o$ and $\ddot{y}_o$ is indicated in Figure 9 wherein the point A corresponds to the point A in Figure 8 and the response acceleration amplitude $\ddot{y}_o$ is indicated by point 1. The coordinate scale for $\ddot{x}$ is at the left side of Figure 9, while the scale for $\ddot{y}_o$ is at the right side. The scale for the response acceleration amplitude is for a component having $2\frac{1}{2}$ percent of critical damping. To be suitable for aircraft service the component must withstand the response acceleration amplitude $\ddot{y}_o$ indicated by point 1 for an indefinitely long duration; i.e., for a minimum of five million cycles.

Since it is not feasible to conduct a laboratory vibration test for five million cycles, it becomes necessary to select a smaller number of cycles and an increased test amplitude. Selecting ten thousand cycles $N = 10^4$ as a reasonable test period, the acceleration amplitude $\ddot{x}_o$ of the vibration test is indicated by point B in Figure 9 and the response acceleration amplitude $\ddot{y}_o$ of the component is indicated by point 2. This applies only if the test frequency has been adjusted to exactly coincide with the natural frequency f_2. If the difference between test and natural frequencies is 1 percent the response acceleration amplitude $\ddot{y}_o$ of the component is indicated by point 3. It is important to note that point 3 represents a lower response than point 1. Consequently, failure of the component may not be expected even though the test is continued indefinitely. If the mismatch of test and natural frequencies is 2 percent, the corresponding response acceleration amplitude $\ddot{y}_o$ is indicated by point 4. Under these circumstances, it is still less likely that failure of the component will occur during the test period. This appears to indicate that it is not practical to conduct vibration tests at discrete frequency intervals because an excessively large number of increments will be involved and because it becomes necessary to adjust the test frequency with unreasonable accuracy.

The alternative to a test at discrete frequencies is one in which the test frequency is continuously varied between limits. This may be illustrated by reference to Figure 11. A hypothetical equipment under consideration is assumed to include two elements whose natural frequencies are f_2 and f_4. The test frequency then is assumed to vary continuously from a minimum of f_o to a maximum f_8. The environment is indicated by line 1 in Figure 11, corresponding to a portion of the environment line b in Figure 8.

Inasmuch as the laboratory test will be conducted for relatively few cycles, the acceleration amplitude $\ddot{x}_o$ for the test is increased with respect to the acceleration amplitude $\ddot{x}_o$ of the environment, as indicated by line 3 in Figure 11. If the environment includes vibration at a frequency f_2 and at an acceleration amplitude $\ddot{x}_o$ corresponding to the maximum expected amplitude represented by line 1, the element whose natural frequency is f_2 will have a response acceleration amplitude $\ddot{y}_o$ at resonance indicated by point 2 in Figure 11. During the laboratory test with increased acceleration amplitude the element whose natural frequency is f_2 will exhibit the response acceleration amplitude $\ddot{y}_o$ outlined by line 4 as the test frequency continuously changes from f_1 to f_3. The value of $\ddot{y}_o$ at the frequency f_2 can be determined from $\ddot{x}_o$ and Figure 5 if the rate of change of the test frequency is very slow. If the rate of change is not slow, $\ddot{y}_o$ may have a lower value because the resonant condition endures only momentarily. Furthermore, the element will experience a very small number of cycles at maximum response acceleration amplitude. It does, however, experience a greater number of cycles at somewhat reduced amplitudes on either side of resonance and certain of those lower cycles may contribute to damage of the element. As compensation, it may be assumed that all of the cycles in region a, of Figure 11 which are 80% or more of the resonant response, experience maximum response to compensate for neglecting the damage contributed by the lower responses in adjacent regions b.

The pattern by which a continuously varying test frequency varies is important. If the test frequency varies too fast, the response amplitude of equipment being tested does not build up as high as predicted by steady-state vibration theory. If the rate of variation of the test frequency is low, the response amplitude builds up approximately to that obtained during steady-state vibration and it is possible to obtain an appreciable number of cycles of vibration at a response close to the maximum. This suggests that care should be exercised in selecting a rate of variation of the test frequency.

This problem has been studied in detail in reference 22 of Appendix III and the results are summarized in Figure 12. This figure illustrates the ratio $\ddot{y}_o/\ddot{x}_o$ for a damped, single-degree-of-freedom system ($Q = 20$) at various rates of change of the test frequency as a function of the dimensionless time r/R. The natural frequency of the vibrating system in cycles per second is indicated by f_n, h represents the rate of change of the test frequency in cycles per second per second, r represents the total number of free vibrations of the system strating at zero time, and R represents the value of r at which the instantaneous forcing frequency equals the natural frequency of the system. A large value of R, therefore indicates a slow rate of change of the test frequency. The relation between f_o, h and R is:

$$h = f_n^2/R \qquad (3)$$

As pointed out previously, significantly great stress cycles occur only at test frequencies approximating the natural frequencies of vulnerable elements. It is desirable that each element of the equipment experience the same number of significant stress cycles during vibration tests independently of the natural frequency of the element. Consequently, the rate of change of the test frequency should increase as the test frequency increases. Referring to Figure 12, this indicates that the dimensionless ratio $\underline{r}/\underline{R}$ should be maintained constant.

It is possible to write an expression as follows for any instantaneous test frequency $\underline{f_1}$ in terms of the minimum test frequency $\underline{f_o}$ and the time $\underline{t_1}$ required to change the test frequency from its minimum value $\underline{f_o}$ to frequency $\underline{f_1}$:

$$f_1 = f_o + \int_0^{t_1} h \, dt \tag{4}$$

This equation may be evaluated numerically by assuming a value for the parameter $\underline{R}$. It is evident from the relation set forth in Figure 12 that the parameter $\underline{R}$ should be maintained relatively large to permit the response amplitude to build up to a significantly high value. A value 1200 is assumed for the parameter $\underline{R}$. By reference to Figure 12, it is observed that the stress in an element at resonance is above 80 percent of the maximum value when $0.990 < \underline{r}/\underline{R} < 1.037$. Since $R = 1200$, $1188 < r < 1245.6$ and the stress is above 80 percent of the maximum value for 57.6 cycles of vibration during each sweep of the test frequency. Inasmuch as the dimensionless ratio $\underline{r}/\underline{R}$ is a constant, the stress will exceed 80 percent of the maximum for any element, independent of its natural frequency, provided the rate of change of test frequency defined by equation (3) is maintained.

The results of a numerical evaluation of equation (4) are given in Table I. At the rate of change of the test frequency indicated by a value $\underline{R} = 1200$, a period of approximately 180 seconds is required to increase the test frequency from 5 to 50 cycles per second, and approximately 18 seconds is required to increase the test frequency from 50 to 500 cycles per second. These periods are noted separately here, and will be maintained separate throughout the discussion because different testing machines probably are required for testing in these two frequency intervals. As the test frequency is increased from 5 to 500 cycles per second according to this pattern, each element with a natural frequency within this range will experience a stress of 80 percent or more of the maximum for 57.6 cycles.

The testing procedure being discussed here contemplates an equal number of stress reversals throughout the frequency range, independent of the test frequency. It is thus necessary that the rate of change of test frequency, $\underline{h}$, increase as the frequency increases. The necessary rate of change of test frequency is obtained from equation (3) by setting the natural frequency equal to the test frequency. The required rate of change of test frequency is set forth in Figure 13a for various values of $\underline{R}$. Figure 13b summarizes the results of calculations similar to Table I for various values of $\underline{R}$, and also the number of cycles of significant stress; i.e., greater than 80% of maximum response. It can be seen that both the number of significant cycles and the sweep time increase in direct proportion to $\underline{R}$. Accordingly it can be concluded that the total time required for a test, with an equal number of stress reversals throughout the frequency range, is independent of $\underline{R}$. The parameter $\underline{R}$ will govern the number of sweeps required to complete the test.

To determine the relation between (1) the number of sweeps between minimum and maximum test frequencies and (2) the applicable test amplitude, it is necessary to define the characteristics of curve I in Figure 9. The actual numerical values which apply to the ordinate scale on Figure 9 are not important because curve I is employed to determine only the ratios of acceleration amplitudes which correspond to certain ratios of cycles to failure. Curve I is assumed to have a relative amplitude of unity at $\underline{N} = 10^3$ cycles to failure and a relative amplitude of 0.38 at $\underline{N} = 5 \times 10^6$ cycles to failure. Based upon a linear relation between these two extreme points, the equation defining curve I in Figure 9 in the interval $10^3 < \underline{N} < 5 \times 10^6$ is:

$$\ddot{x}_0 = 1.00 - 0.167 \log_{10}(N/10^3) \tag{5}$$

In equation (5), $\underline{N}$ is any value of cycles to failure between 1000 and 5,000,000 while $\ddot{x}_0$ is the ordinate value corresponding to the selected value of $\underline{N}$, based upon a relative ordinate of unity at 1000 cycles and 0.38 at 5,000,000 cycles.

Although infinite life is a practical objective, the analysis predicts failure after 5×10^6 cycles of vibration at a relative amplitude of 0.38 in Figure 9. An amplitude exaggeration factor will now be determined to find the relative amplitude necessary to cause similar failure in a substantially fewer number of cycles. Assuming several representative values for N, corresponding values of $\ddot{x}_0$ are calculated from equation (5) and are set forth in the second column of Table II. The ratios of these values of $\ddot{x}_0$ to the value 0.38 which applies to $\underline{N} = 5 \times 10^6$ are set forth in column three of Table II. These ratios are, in effect, exaggeration factors by which the test amplitude must be increased to cause failure in $\underline{N}$ number of cycles.

As indicated above, the tests being considered here contemplate a sweep frequency procedure which causes the number of cycles of significant stress per sweep indicated in Figure 13b for various values of the parameter R. To attain the total number of cycles set forth in the first column of Table II, it is necessary to employ a number of sweeps equal to the ratio N divided by significant cycles from Figure 13b. This ratio (for a value of R = 1200) is then multiplied by 178 and 17.8 seconds, respectively, to obtain the times set forth in columns 4 and 5 of Table II. The total testing time is the sum of the times set forth in columns 4 and 5, and is given in column 6.

A laboratory test is now established by selecting an appropriate line in Table II. Various considerations must be kept in mind in making this selection. It is desirable that the exaggeration factor be maintained as small as possible because the response amplitude of the equipment under test may be affected by non-linearities introduced by the high amplitude testing. On the other hand, it is desirable to maintain the total testing time small for experience in conducting tests. The best compromise between these opposing considerations, from the alternatives set forth in Table II, appears to embody a test in which each element receives 4000 cycles of stress reversal ($N = 4 \times 10^3$). This gives a testing procedure involving approximately 4 hours of vibration testing in which the testing frequency is varied continuously between 5 and 50 cycles per second in a period of 178 seconds, and between 50 and 500 cycles per second in a period of 17.8 seconds. This involves the number of sweeps of test frequency variation between 5 and 500 cycles per second indicated in Figure 13c for various values of the parameter R. This involves an exaggeration factor of approximately 2.37. The environmental condition embodying a displacement amplitude of 0.030" is then exaggerated to a displacement amplitude of 0.071" for laboratory testing in the frequency range of 5 to 50 cycles per second. In a similar manner, the environmental condition of 4 g acceleration amplitude is exaggerated to an acceleration amplitude of 9.5g for laboratory testing in the frequency range of 50 to 500 cycles per second.

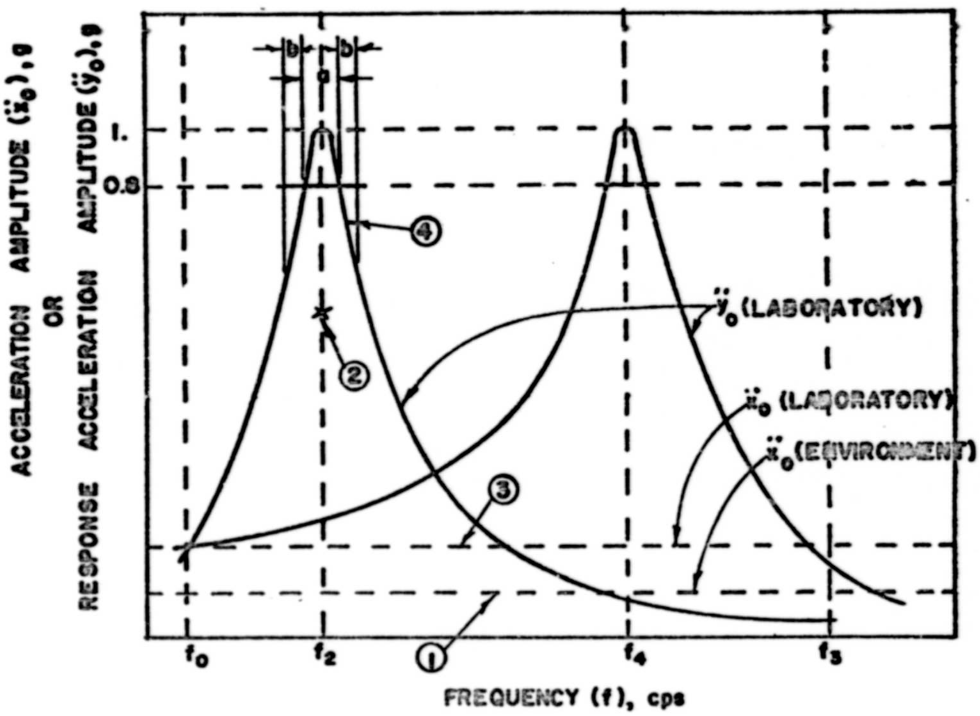


FIGURE 11. RELATION BETWEEN ACCELERATION AMPLITUDE $\ddot{x}_0$ AND RESPONSE ACCELERATION AMPLITUDE $\ddot{y}_0$ FOR CONDITION OF CONTINUOUSLY VARYING TEST FREQUENCY.

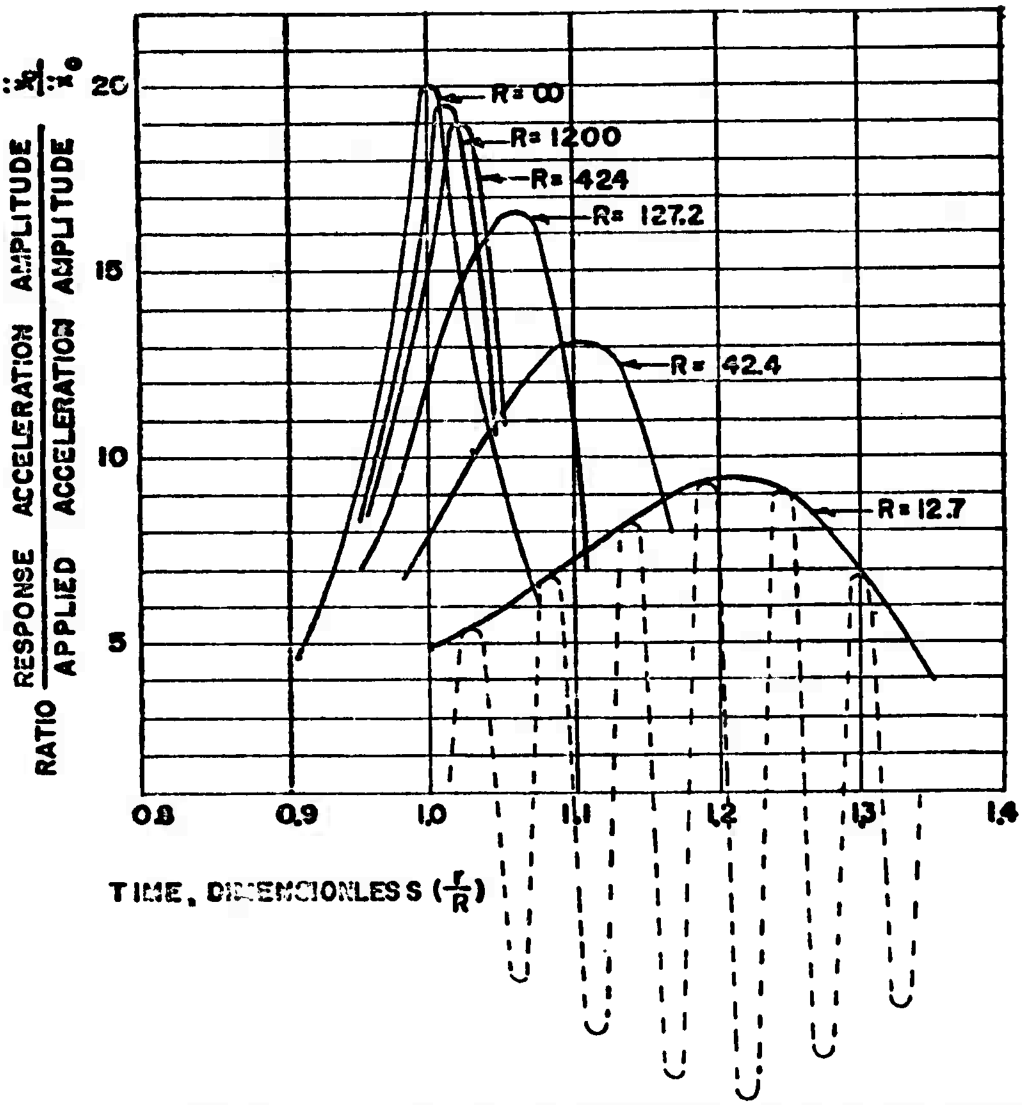


FIGURE 12. RATIO OF RESPONSE TO APPLIED ACCELERATION AMPLITUDES FOR SYSTEM WITH $Q = 20$ WHEN TEST FREQUENCY VARIES CONTINUOUSLY.

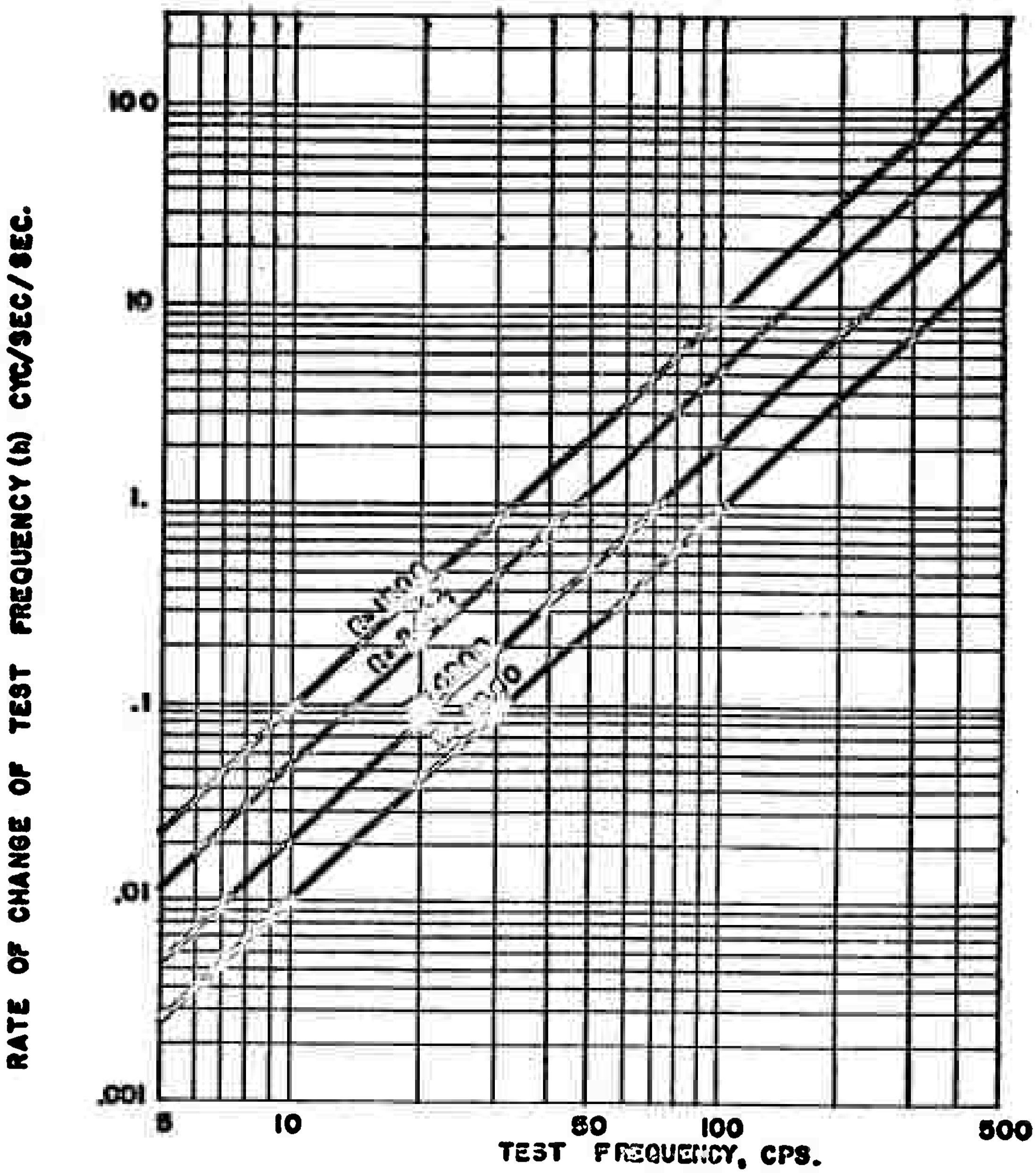


FIGURE 13a. RATE OF CHANGE OF TEST FREQUENCY FOR STEADY-STATE TEST FOR VARIOUS VALUES OF THE PARAMETER R.

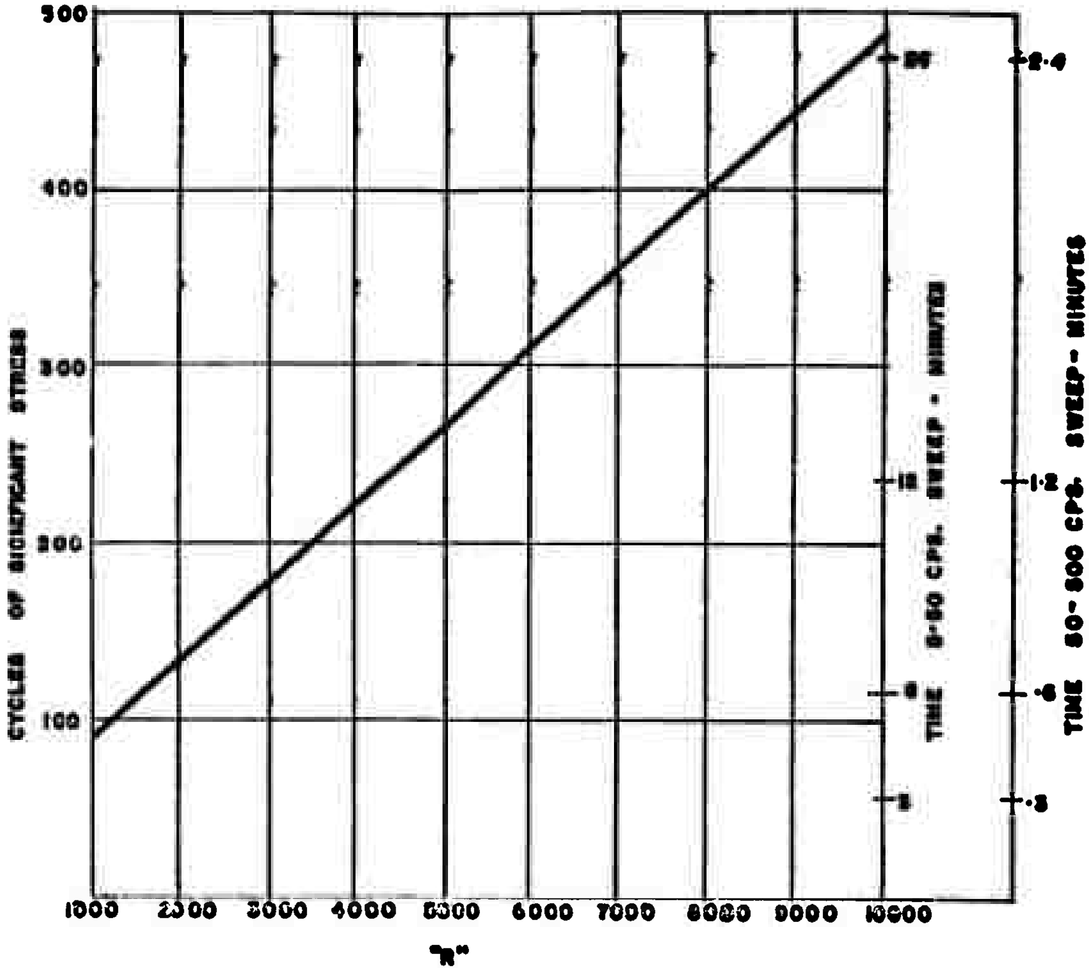


FIGURE. 13b. SWEEP TIME AND CYCLES OF SIGNIFICANT STRESS AS A FUNCTION OF THE PARAMETER "R"

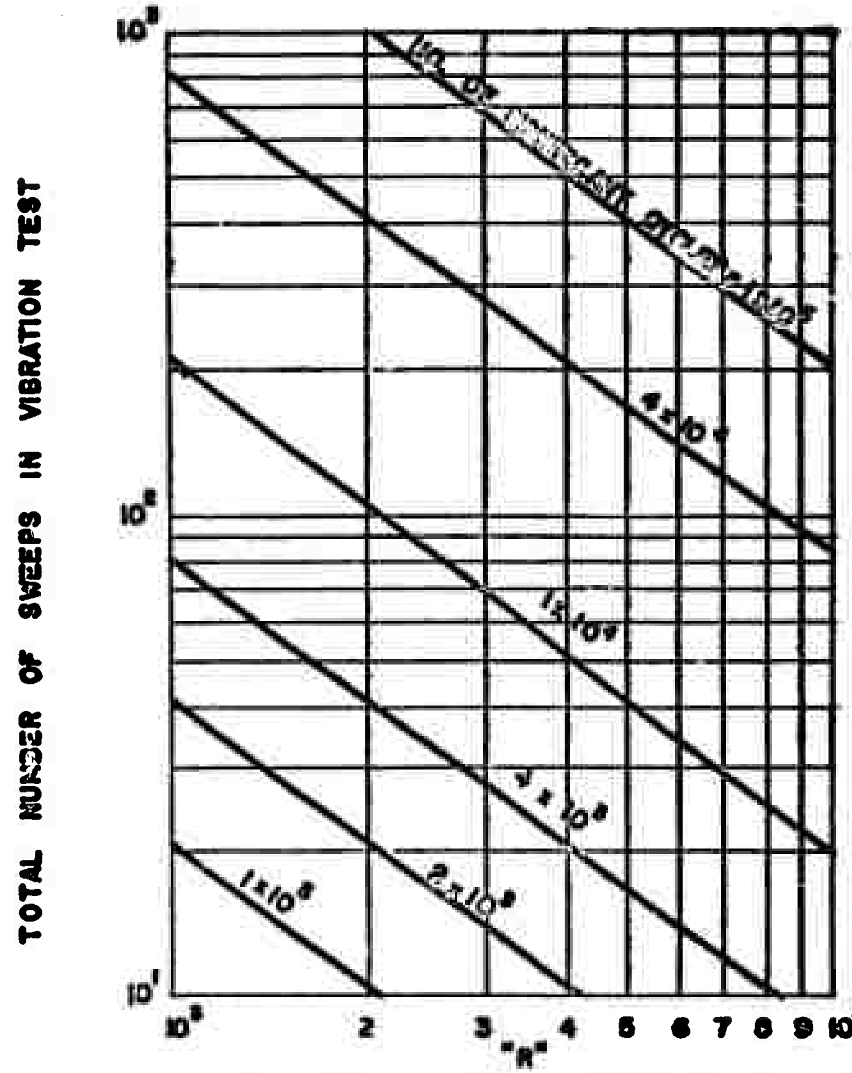


FIGURE 13C. TOTAL NUMBER OF SWEEPS IN VIBRATION TEST FOR VARIOUS VALUES OF PARAMETER R AND SIGNIFICANT CYCLES.

TABLE I. NUMERICAL EVALUATION OF SWEEP TIME FOR 5 TO 500 CPS.

For R = 1200

Number of significant cycles per sweep = 1200 (1.038-0.990)

= 57.6

$$f_1 = f_0 + \int_{t_0}^{t_1} h_{AVE.}\,dt = f_0 + \left(\frac{f_1^2 + f_0^2}{2R}\right)\int_{t_0}^{t_1} dt = f_0 + \left(\frac{f_1^2 + f_0^2}{2R}\right)(t_1 - t_0)$$

$$\Delta f = f_1 - f_0 \quad \& \quad \Delta t = t_1 - t_0$$

$$\Delta t = \frac{\Delta f}{\left(\frac{f_1^2 + f_0^2}{2R}\right)}$$

f_1 cps.	f_0 cps.	$h_{ave.} = \frac{f_1^2 + f_0^2}{2R}$ cyc./sec.	$\Delta t = \frac{\Delta f}{h_{ave.}}$ sec.	t sec.
10	5	.052	96.2	
20	10	.208	48.	
30	20	.541	18.5	
40	30	1.04	9.6	
50	40	1.70	5.9	178.2
100	50	5.20	9.62	
200	100	20.8	4.8	
300	200	54.1	1.85	
400	300	104.	.96	
500	400	170.	.59	17.8

TABLE II. EVALUATION OF TOTAL VIBRATION TEST TIME FOR VARIOUS EXAGGERATION FACTORS

N	$\ddot{x}_o$	Exaggeration Factor $\frac{\ddot{x}_{c_n(n=N)}}{\ddot{x}_{c_n(n=5 \times 10^8)}}$	Time for 5-50 cps. Test	Time for 50-500 cps	TOTAL TIME	
Cycles	-		Minutes	Minutes	Hours and Minutes	
10^3	1.00	2.63	52.6	5.3	57.9 min.	57.9 min.
2×10^3	0.95	2.50	105.2	10.5	115.7 min. = 1 hr.	55.7 min.
4×10^3	0.90	2.37	210.4	21.0	231.4 min. = 3 hr.	51.4 min.
10^4	0.83	2.19	526.	52.6	578.6 min. = 9 hr.	38.6 min.
4×10^4	0.73	[illegible]	2104.	210.	2314. min. = 38 hr.	34 min.
10^5	0.67	1.75	5260.	526.	5786. min. = 96 hr.	26 min.

VIBRATION TEST SPECIFICATION

The preceding analysis presumes failure of the equipment as a result of repeated stressing of structural members. The test embodies an amplitude greater than that encountered in the environment for the purpose of causing structural failure in a relatively short period of time. Under the assumed conditions, vibration of this magnitude never occurs in service. Consequently, it should not be required that the equipment under test be functionally operative during this exaggerated test condition. The preceding endurance test should be supplemented by a scanning test at an environment level corresponding to that actually experienced in service, and the equipment should be functionally operative under such conditions.

In summary, the recommended vibration tests are as follows:

(a) A scanning test at a displacement amplitude of 0.030" peak-to-peak throughout a frequency range of 5 to 50 cycles per second, and at an acceleration amplitude of 4g throughout a frequency range of 50 to 500 cycles per second. This test is intended not to investigate the structural integrity of the equipment but only to determine that it operates satisfactorily. This vibration test is considered to simulate the expected maximum environment.

(b) Vibration at a displacement amplitude of 0.070" peak-to-peak throughout a frequency range of 5 to 50 cycles per second for an elapsed time of 210 minutes. This is a sweep frequency test in which the test frequency is continuously varied at any of the rates set forth in Figure 13a. In addition, vibration at an acceleration amplitude of 9.5g throughout the frequency range of 50 to 500 cycles per second for an elapsed time of 21 minutes. This is also a sweep frequency test with any of the rates of change of test frequency set forth in Figure 13a. This is a test to investigate structural strength. The equipment should not be required to function during the test but should remain undamaged and fully operative at the conclusion of the test.

The data defining the vibration environment in aircraft make but few distinctions with regard to direction of motion. The directions must be assumed to be completely random, with the result that the environment defined in Figures 2 and 3 is considered applicable to each of three coordinate axis independently. In many equipments, there is coupling between two or more directions of motion. In other words, vibration that is applied along one axis may cause certain structures to vibrate in the direction of one or more other axis. For this reason, vibration of one equipment in each of three directions for a period of three hours each may subject certain structures to more than four hours of vibration. It is recommended, therefore, that the vibration test sequence apply equally to vibration along each of three coordinate axis and that separate or rebuilt equipments be used for each direction.

As described in complete detail on the preceding pages, the procedure followed in arriving at a laboratory test involving steady-state vibration consists of three discrete steps. These may be summarized as follows:

(a) The limits of the probable maximum severity of the environment in terms of steady-state vibration must be defined.

(b) The principles governing the relation between failure after a relatively long period of mild vibration and failure after a relatively short period of severe vibration must be established.

(c) Using the environment established in (a), the principles established in (b) must be applied by selecting an exaggerated test condition which will cause a representative type of failure within a reasonable testing period.

The testing routine set forth above for conditions of steady-state vibration is the result of applying this procedure to data defining the environment in the establishment of a laboratory test. The validity of the routine is dependent upon the authenticity of the data which defines the environment. Further comments regarding data are set forth in the section of this report entitled "Conclusions". Data defining vibration environments make few distinctions with regard to simultaneous environments of humidity, sand and dust, extreme temperatures or other environments. In general these latter environments must be considered as additive to vibration or shock environments and should be tested simultaneously.

CONCEPTS OF DAMAGE AS A RESULT OF SHOCK

This section of the report discusses the analysis of records of acceleration as a function of time used to define conditions of shock or transient vibration. These records are the oscillograms mentioned earlier in the report. In analyzing the effect of vibration upon mechanical systems, there is a fundamental distinction between procedures for considering transient and steady-state vibration. In steady-state vibration, it is assumed that all transient effects occur at the natural frequency of the system and are ultimately damped out. The motion of the system then takes place entirely at the frequency of the disturbing vibration. This frequency is referred to as the forcing frequency.

In shock or transient vibration, the equipment which is subjected to the vibration is caused to vibrate in a mode that includes both forced and natural frequencies. In the foregoing analysis of an equipment subjected to steady-state vibration, it was assumed that damage results primarily from a resonant condition at the forcing frequency. In a sense, the same consideration applies to transient vibration except that the forcing frequency is not as well defined as in steady-state vibration. Consequently, there are no established standards of frequency to use in setting up a laboratory test. An entirely different approach is thus needed in adopting a criterion of damage.

As pointed out previously in connection with steady-state vibration, a primary cause of certain types of failure is excessive stress in structural members. This stress tends to be proportional to the acceleration experienced by such members. From a hypothetical standpoint, it is possible to predict the stresses in the structural members by attaching an accelerometer to each element of the equipment being tested and multiplying the measured acceleration by the mass to obtain the force acting upon the member. If the strength of each such element were known, it would then be possible to predict from the acceleration measurements whether the elements were close to failure as a result of the test. Although such a procedure is possible in a hypothetical sense, it is not practically feasible because the vulnerable elements tend to be small and inaccessible whereas applicable accelerometers tend to be relatively large.

Even though this suggested procedure is impractical, the theory suggests an approach to evaluating the severity of shock or transient vibration in terms of possible damage to an equipment. Assume now that each equipment is comprised

of many structural elements having characteristic natural frequencies assumed to fall within a certain frequency range. A single equipment cannot include elements of all natural frequencies. If many equipments are taken as a group, however, it is probable that at least one element of each discrete natural frequency within the accepted frequency range will be found in one of the equipments in the group.

To a first approximation, the maximum acceleration experienced by a structural element of an equipment is a function of only the natural frequency of the element. Consequently, a simulated equipment may be constructed in any convenient form, such as the base equipped with cantilever beams illustrated in Figure 14. Each of the cantilever beams of this simulated equipment has a different natural frequency, and the range of natural frequencies to be studied may be changed at will by varying the lengths of the cantilever beams. If it is desired to determine the required strength of the elements of an equipment which must withstand a certain shock, the simulated equipment illustrated in Figure 14 may be subjected to the shock and the maximum deflection of each beam noted. The maximum acceleration of each beam is then calculated from the recorded maximum deflection. The values of maximum acceleration determined in this manner are used in conjunction with the masses of elements of the actual equipment having corresponding natural frequencies as the cantilever beams to determine the dynamic forces acting on each element as a result of the shock. Theoretically, this simulated equipment may be subjected to all possible occurrences of shock or transient vibration and the maximum acceleration of each cantilever beam noted. With this information for each natural frequency, the designer may design the actual equipment so that each element has at least the strength indicated by the maximum acceleration of the cantilever beam having the same natural frequency.

The use of a mechanical instrument to obtain the information outlined in the above paragraph tends to be cumbersome. Equivalent results can be obtained by electrical analogy. Electrical circuits can be assembled in such a way that they respond to an input voltage in the same manner as a mechanical structure responds to an applied shock or transient vibration. The details of a suitable analog computer, together with a specially constructed function generator, are described in Appendix II. With this analog computer, the natural frequencies and damping coefficients of the systems under investigation can be varied readily by adjusting the constants of the electrical circuits. It is then possible to obtain electrical responses which, by analogy, can be converted to values of maximum acceleration experienced by mechanical structures having known characteristics. Each shock or transient vibration is then defined in terms of maximum response acceleration. This may be

expressed as a curve showing maximum response acceleration as a function of natural frequency of element for a single shock motion; for a group of shock motions, an envelope may be drawn to encompass the curves for the individual shock motions.

This concept of severity, together with the method of expressing equivalents of shock motions, is soundly endorsed in the technical literature. References 25, 33, and 99 in the Bibliography, Appendix III, are significant with respect to this concept. This approach is satisfactory where the response embodies a relatively high value of acceleration for a single cycle and where the remainder of the response embodies relatively small values of acceleration. Under these conditions, only the maximum acceleration tends to be significant. The present analysis, however, is concerned with shock and transient vibration occurring in aircraft as a result of landing, gun fire, air buffeting and other disturbances having many repetitions. As a consequence, it may not be possible to express the results in terms of a single cycle or a few cycles of stress reversal.

As an example of the different types of responses that may be obtained, a record of acceleration as a function of time resulting from the landing of a type AT-11 airplane and several typical responses thereto are shown in Figure 15. The acceleration as measured on the airplane is the lower trace in each of the four oscillograms, the differences in calibration factors accounting for the different appearances. This trace, suitably expressed in terms of voltage as a function of time was fed into the analog computer, and the responses of several systems of different natural frequencies were obtained. These responses are reproduced as the upper traces of the several oscillograms in Figure 15.

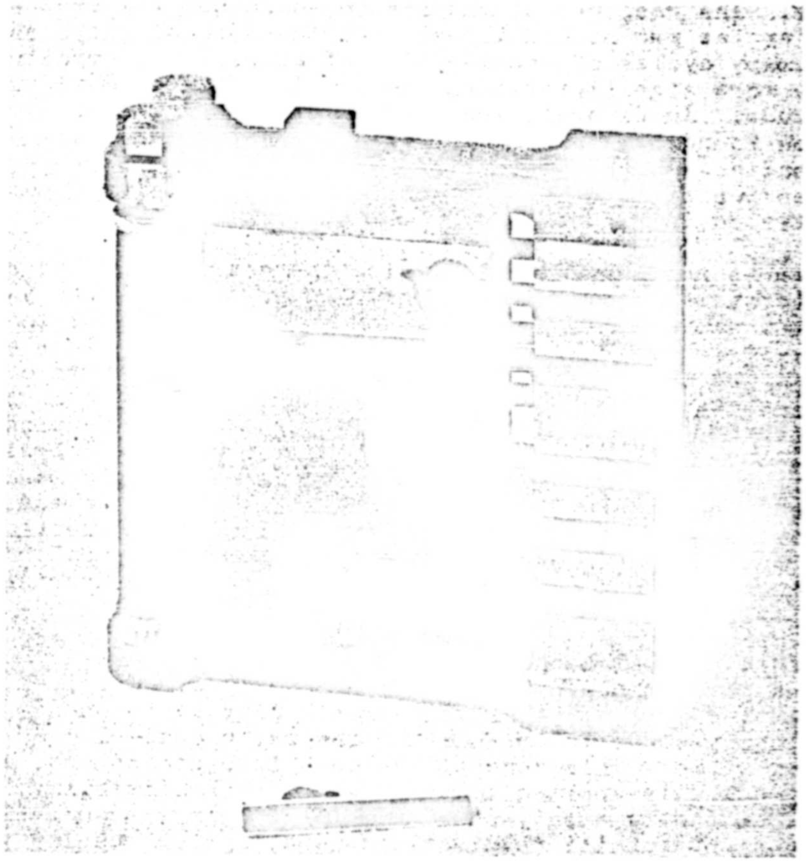
The response set forth in Figure 15 (A) for a system having a natural frequency of 10 cycles per second. This low frequency system fails to respond to the high frequency components in the input, and the response is an irregular record having predominantly low frequency components. At the other extreme of the frequency spectrum, the responses of systems having natural frequencies of 110 and 200 cycles per second are shown in Figures 15 (C) and 15 (D), respectively. The general shapes of these responses are similar to the shape of the input but have superimposed thereon transient vibration at the natural frequencies of 110 and 200 cycles per second. The amplitude of this superimposed vibration is relatively small. In Figure 15 (B), the upper trace is the response of a system having a natural frequency of 33 cycles per second.

In the response of Figure 15 (B), the shape of the input is almost obscured, and the predominant characteristic of the response is a transient vibration having a natural frequency of 33 cycles per second and a relatively great amplitude. This frequency is the natural frequency of the

system and represents a sort of resonance. This resonance occurs because the input vibration has a prominent component at a frequency of approximately 33 cycles per second, even though this component of the input is not steady-state in nature. The response of the system whose natural frequency is 33 cycles per second illustrates the type of response in which many cycles of acceleration of appreciable amplitude may be more significant than the acceleration of maximum amplitude. In general, the acceleration measured on aircraft will be found to be irregular with certain predominant frequencies. The responses of systems covering a range of frequencies will be similar generally to the several responses illustrated in Figure 15.

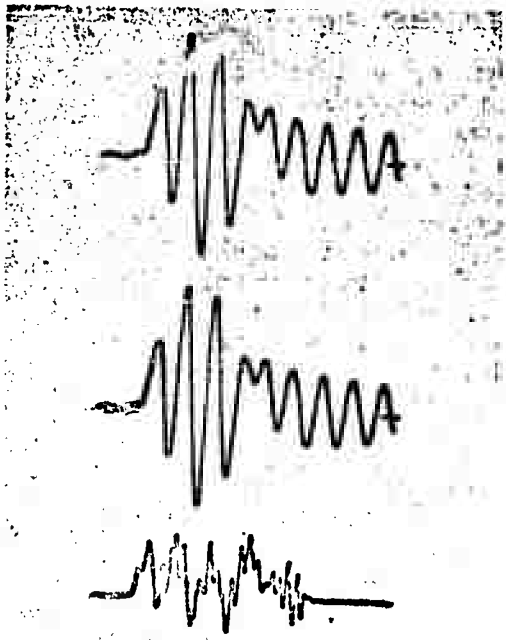
It is necessary in describing the characteristics of an elastic system to define not only the natural frequency but also the damping. The response of an elastic system to steady-state or transient vibration is greatly influenced by the damping of the system. In the preceding analysis of steady-state conditions, the environment is defined in terms of measured parameters and is not a function of the properties of systems subjected to the environment. In the analysis of transient conditions, the environment is defined in terms of the response of elastic systems to the measured parameters. The environment can be defined completely only by setting forth the responses of elastic systems having natural frequencies and degrees of damping encompassing the range expected to be encountered in equipments.

In classical mechanics, damping is defined as a percentage of the damping in a critically damped system. If the damping is small, the transmissibility at resonance is numerically equal to one-half of the reciprocal of the damping ratio. This concept has come into common use, and the symbol Q is applied to the maximum transmissibility at resonance during steady-state vibration. In the analysis of transient conditions discussed here, systems having values for Q of 10, 20, and 50 are considered. The selection of an appropriate value for Q is difficult. The damping in structural members tends to increase with an increase in strain. If the vibration amplitude is small, it may be expected that large values of Q will be encountered, and the maximum value of 50 used here may seem too low. For the relatively large deflections embodied in the transient conditions being studied here, however, strains tend to be large, and it is believed that a range of 10 to 50 for the parameter Q is representative of these conditions.

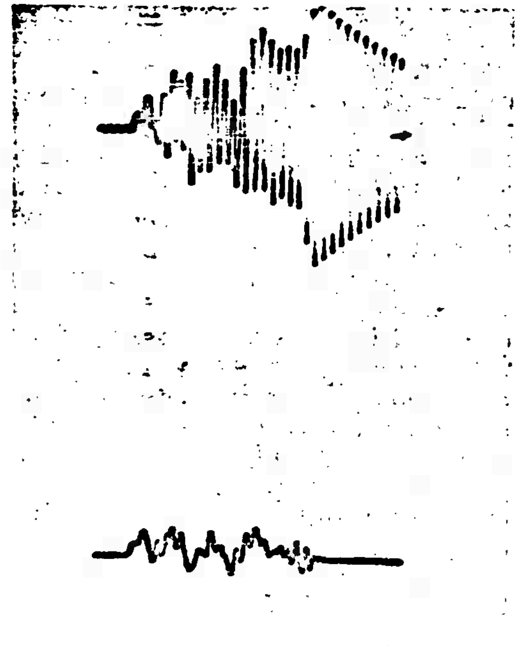


Courtesy of The David Taylor Model Basin,
USN Photo No. NP21-48384.

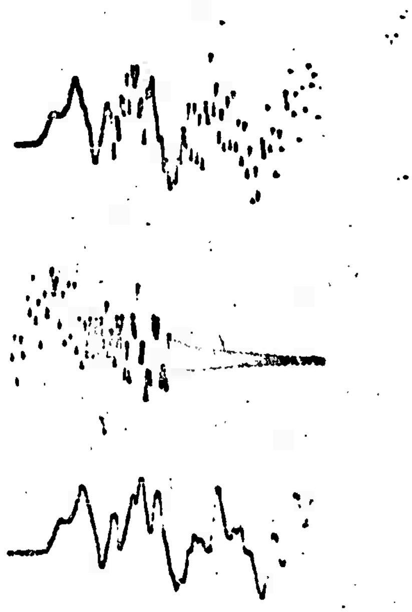
Figure 14. Simulated equipment, consisting of base and several cantilever beam systems of different natural frequencies.



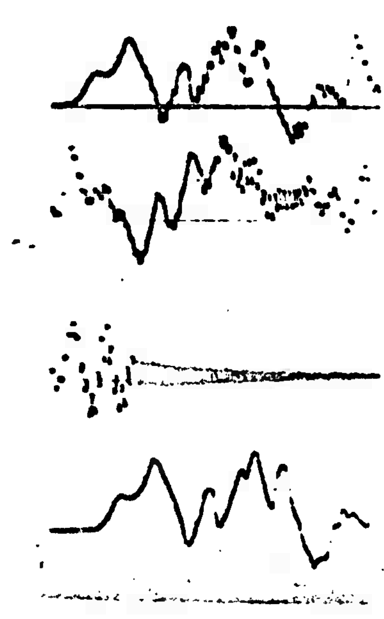
(A) $f_n = 10$ cps., $Q = 50$



(B) $f_n = 33$ cps., $Q = 50$



(C) $f_n = 110$ cps., $Q = 50$



(D) $f_n = 200$ cps, $Q = 50$

Figure 15. Response acceleration as a function of time for systems having natural frequencies of 10, 33, 110, and 200 cps. when subjected to landing shock measured on Type AT-11 airplane.

CUMULATIVE DAMAGE IN FATIGUE

In the establishment of laboratory tests to simulate the environmental conditions encountered in the operation of aircraft, cognizance must be taken of the fact that an aircraft experiences many landings in its lifetime. As indicated in Figure 15, elastic structures carried in such aircraft experience several stress reversals per landing. It may be deduced intuitively that the cumulative effect of these several cycles of stress reversal may be of great importance if the stress magnitude at each cycle is approximately the same. On the other hand, if the response exhibits one cycle of stress of relatively great magnitude, the cumulative effect of other cycles may be negligible if the magnitudes of these other cycles are substantially less than the magnitude of the maximum stress. It cannot be determined by an inspection of the response patterns whether the cumulative effect of the cycles of lower stress is important. Consequently, the two types of analysis have been carried out concurrently, and the results thereof compared. The first part of the analysis is devoted to a consideration of the cumulative effect of cycles of stress having a magnitude somewhat less than the maximum. This is then compared with the corresponding results obtained by considering only the maximum stress encountered during a single landing.

The technical literature includes numerous papers setting forth the fatigue or endurance properties of materials. The tests generally are conducted under such circumstances that the maximum stress is constant at each cycle of stress reversal. The responses illustrated in Figure 15 do not meet these requirements for two principal reasons as follows:

(a) A different maximum stress is encountered at each succeeding cycle of stress reversal. An analysis of endurance strength under conditions involving a stress pattern having something in common with this was carried out initially by M. A. Miner who reported the results of his analysis in reference 26. The problem considered by Miner involved the application of many cycles of stress having the same maximum value at each cycle, followed by many cycles at a different maximum stress, etc. Although the hypothesis formulated by Miner does not contemplate the present situation in which the maximum stress is different at each succeeding cycle of stress reversal, it appears quite general. The assumption is made here that the hypothesis may be extended to the type of stress pattern found here, even though the paper by Miner does not establish the validity of this assumption.

(b) In the oscillograms set forth in Figure 15, the mean value of response acceleration is generally

different than zero. In the conventional type of test for endurance strength, a tensile stress is followed one half cycle later by a compressive stress of equal magnitude. There is considerable information in the technical literature on fatigue tests in which the value of one of these stresses exceeds the other, thereby introducing a mean stress not equal to zero. It is possible to make a correction to a condition of zero mean stress, a correction which has become known as the Goodman correction. The validity of the Goodman correction has been established only for conditions of a regular stress pattern, and has not been demonstrated to apply to the type of stress pattern investigated by Miner. It is possible, however, by making several assumptions to apply the Goodman correction to an hypothesis of the transient problem based upon the approach proposed by Miner. A few sample calculations revealed that the results are not modified appreciably by introducing the Goodman correction for mean stress. Consequently, all of the ensuing calculations are based upon the assumption that only the maximum stress is significant, and that the problem should be handled as if the mean stress were zero.

Reference is now made to the conventional fatigue or endurance curve initially illustrated in Figure 6 and essentially reproduced in Figure 16 to serve as a basis for discussion of Miner's hypothesis. This curve shows a typical relation between maximum stress $\underline{S}$ and number of cycles $\underline{N}$ to failure, when the test is conducted under such conditions that the maximum stress at each cycle is constant. The hypothesis formulated by Miner is concerned with the degree of damage caused when a specimen is subjected to $\underline{n_1}$ cycles of stress reversal at a maximum stress $\underline{S_1}$, $\underline{n_2}$ cycles of stress reversal at a maximum stress $\underline{S_2}$, and $\underline{n_3}$ cycles of stress reversal at a maximum stress $\underline{S_3}$. Referring to Figure 16, Miner's hypothesis states that failure of the specimen should not occur by fatigue if

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} \cdots\cdots < 1 \qquad (8)$$

On the other hand, if

$$\frac{n_1'}{N_1'} + \frac{n_2'}{N_2'} + \frac{n_3'}{N_3'} + \cdots\cdots > 1 \qquad (9)$$

failure of the specimen may be expected to occur.

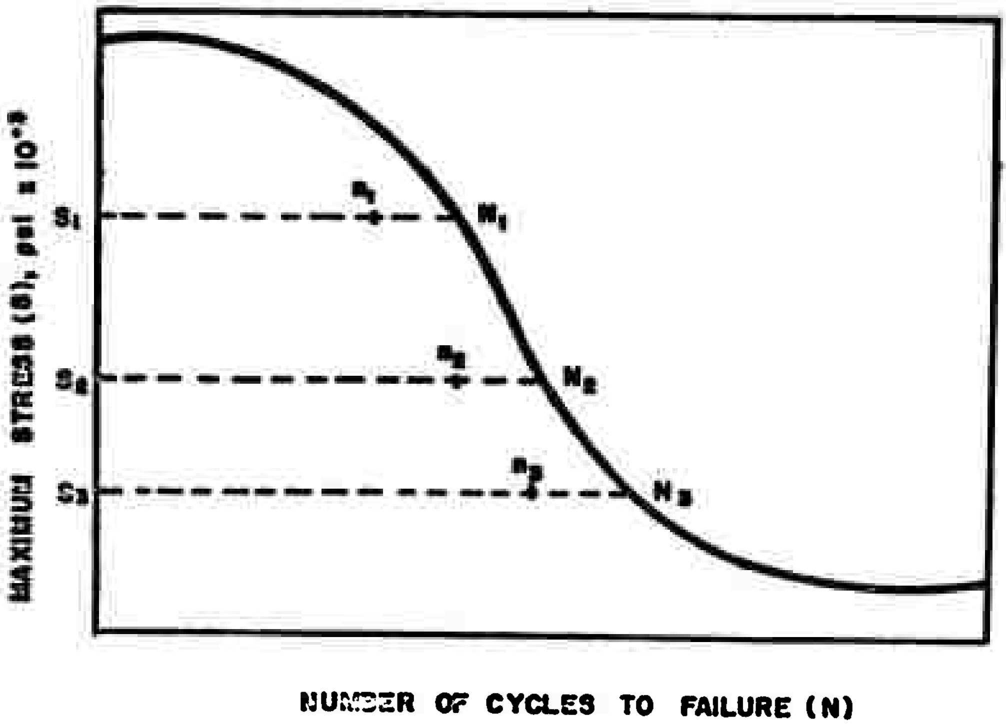


FIGURE 16. TYPICAL STRESS-CYCLE DIAGRAM SHOWING RELATION OF PARAMETERS IN HYPOTHESIS OF CUMULATIVE DAMAGE IN FATIGUE FORMULATED BY M.A. MINER

CONCEPT OF RESPONSE SURFACE TO EVALUATE SHOCK

Adopting the analogy between stress and acceleration (see page 16), the hypothesis is presented that the ability of elastic members to withstand transient vibration may be determined by counting the number of cycles at each acceleration level embodied in the response and applying the theory formulated by Miner. It is convenient now to adopt a procedure for describing the response acceleration of many elements having different natural frequencies, the excitation or input being a single record setting forth acceleration as a function of time. The three parameters which define the response are the natural frequency $\underline{f_n}$ of the element, the acceleration amplitude $\ddot{y}_o$ embodied in the response, and the number of cycles $\underline{n}$ at each response acceleration amplitude. These three parameters may be combined to define the response surface illustrated in Figure 17. This surface describes the response of many systems of different natural frequencies to a single input acceleration and should not be confused with the failure surface for steady vibration set forth in Figure 8.

To illustrate the nature of the surface shown in Figure 17, a number of planes are indicated for various elements having discrete natural frequencies $\underline{f_n}'$, $\underline{f_n}''$ and $\underline{f_n}'''$. To determine whether failure of one of these elements is likely to occur, the plane as represented by the applicable crosshatched area is compared with the endurance curve set forth in Figure 16. This requires that the relation between $\underline{S}$ and $\ddot{y}_o$ be established, as pointed out previously. Applications of Miner's hypothesis to this comparison, in a manner to be hereinafter described, will predict the expected life of each element whose natural frequency is known and for which there is a response surface as illustrated in Figure 17.

An equipment may be considered to consist of an assembly of component structures, each with its own characteristics, and to be defined if the characteristics of the component structures are defined. For purposes of idealizing the equipment, each structure may be assumed to be a single-degree-of-freedom system with linear elasticity and damping. Each system may then be defined in terms of its natural frequency and its damping capacity. The natural frequency is commonly expressed in cycles per second, and the damping capacity may be expressed in terms of a dimensionless damping parameter $\underline{Q}$ which indicates the maximum transmissibility at resonance during a condition of steady-state vibration.

The three dimensional response surface shown in Figure 17 is better adapted to qualitative than quantitative presentation. Numerical data can be recorded more conveniently in two dimensional plots of response acceleration amplitude as a function of number of occurrences, one plot being drawn for each discrete value of natural frequency $\underline{f_n}$ and damping

parameter Q. To facilitate the counting of occurrences, it is convenient to establish discrete increments of response acceleration amplitude $\ddot{Y}_o$, preferably integral or integral fractional values of the acceleration due to gravity. The results may then be presented numerically in block diagram form, as illustrated in Figure 18 for one particular value of natural frequency and damping parameter. Several block diagrams for different natural frequencies but one damping parameter combine to form the response surface shown in Figure 17. Where different values of the damping parameter Q are involved, a discrete response surface exists for each value of Q.

A number of oscillograms showing the time history of acceleration as measured on aircraft during landing and other shock conditions have been made available to Contractor. Several of these oscillograms have been selected for analysis. The criteria used in selecting the oscillograms were (1) that they embody apparently the most severe conditions among the oscillograms available and (2) that the characteristics of the selected oscillograms be as diverse as possible so that the group selected would represent all possible varieties of shock motions. The selected oscillograms are reproduced as insets to the various Figures in Appendix I.

The analysis made on these selected oscillograms includes the determination by electrical analogy of the response acceleration of simple systems having a range of natural frequencies and damping parameters. The results obtained are of the type set forth in Figure 15, wherein the time history of the response acceleration of each system is recorded. The response accelerations are recorded on block diagrams on which response acceleration amplitude $\ddot{Y}_o$ is plotted as a function of number of occurrences, as shown in Figure 18 for a typical case. The parameter b in Figure 18 may be an integer or a fraction. Appendix I to this report includes data on the oscillograms selected, together with block diagrams showing response acceleration amplitude as a function of a number of occurrences for each oscillogram. A block diagram is included for each value of natural frequency f_n and damping parameter Q.

In equations (8) and (9) representing the Miner theory of cumulative fatigue damage, N is the number of cycles to failure obtained from the conventional stress-cycle curve. For stresses below the endurance limit of the material, N is equal to infinity, and the corresponding term drops out of the equation. The typical stress-cycle curve can be idealized as shown in Figure 19 by introducing the following considerations mentioned previously:

(a) The stress-cycle curve is idealized as a horizontal line for values of N less than 10^3 and greater than 5×10^6, and a straight inclined line between these

values of N. The ordinate at $N = 5 \times 10^6$ is 38 percent of the ordinate at $N = 10^3$. The basis for this idealization is explained on page 26 in connection with Figure 9.

(b) By analogy between stress in the structure and response acceleration, as discussed on page 55, the dimensions of the ordinate may be changed to response acceleration amplitude. The numerical relation between maximum stress and response acceleration amplitude remains undetermined at this time to be established later.

This report develops the thesis that a shock is defined, not by parameters which are determined by measuring the environment, but rather by the responses of ideal systems to the shock motion defined by these measured parameters. Under this thesis, each shock is defined by a series of response surfaces of the type illustrated in Figure 17, there being one surface for each value of the damping parameter Q. If several shock motions are being compared, the general level of the several surfaces, the value of Q being maintained constant, indicates the relative severity of the respective shock motions. A peak in a response surface at a particular value of natural frequency f_n generally indicates that the shock motion includes a pronounced vibration at a frequency corresponding to the peak in the response surface. Inasmuch as different shock motions generally embody different frequencies, the response surfaces for these shock motions tend to intersect because the peaks occur at different frequencies. In other words, the peaks of one response surface may be aligned vertically with the valleys of another response surface. An envelope of the most severe conditions representative of all shock motions being considered is a resultant response surface drawn through the peaks of the individual response surfaces, all such surfaces being for the same value of damping parameter Q. A resultant response surface is thus obtained for each discrete value of damping parameter Q.

One of the primary problems in the establishment of a laboratory test to simulate transient environmental conditions is the selection of the most severe environment for use as a basis of simulation. As shown by the block diagrams in Appendix I, certain of the shock motions excite the greatest response at one frequency, while others of the shock motions excite the greatest response at a different frequency. An effective laboratory test must take cognizance of the most severe conditions in general, as defined by the resultant response surface for the series of operating conditions under consideration. This surface may be represented in block diagram form, as shown in Figure 20 for all of the shock motions included in Appendix I. The data in Figure 20 were obtained by first obtaining block diagrams similar to

Figure 18 for each value of natural frequency and damping parameter for each of the shock motions being considered. These block diagrams comprise Appendix I. The resultant block diagrams in Figure 20 are plotted by taking the largest number of occurrences from Appendix I for the respective values of natural frequency and damping parameter. The blocks were omitted for small values of Y_o because they do not contribute to damage as a result of fatigue.

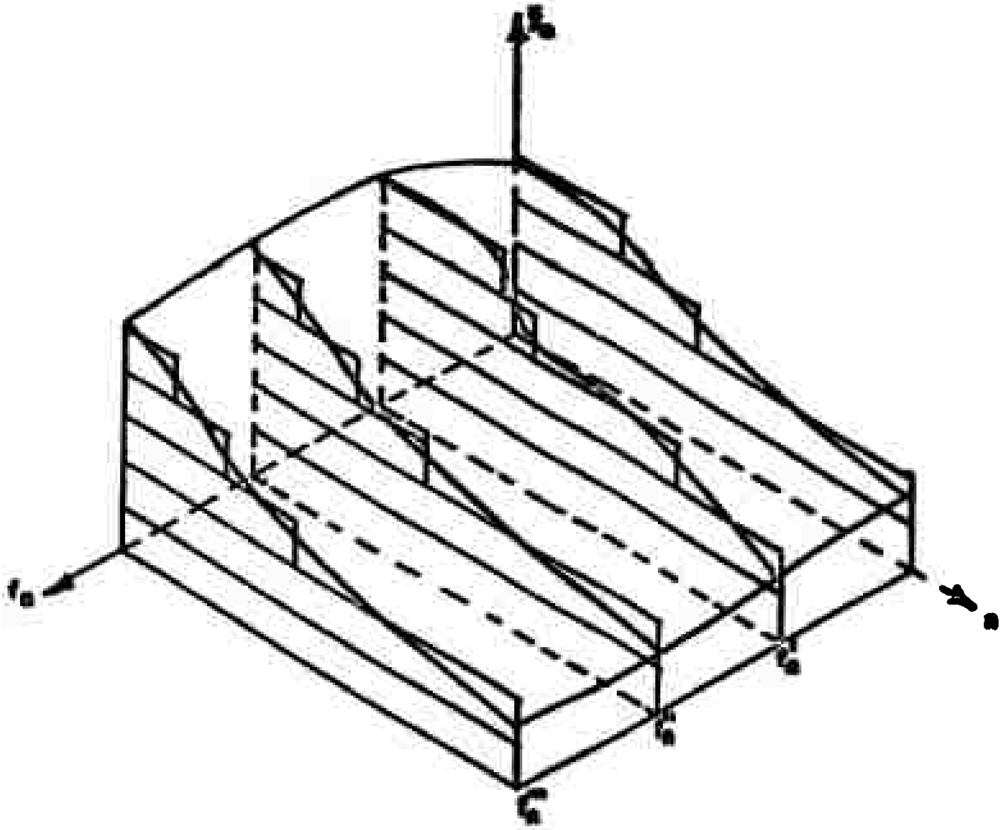


FIGURE 17. TYPICAL RESPONSE SURFACE.

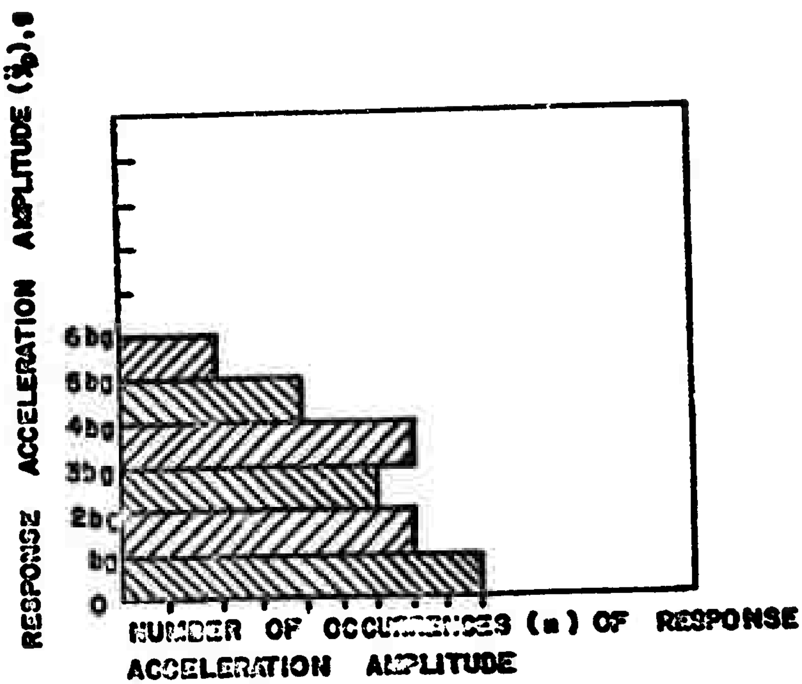


FIGURE 18. TYPICAL BLOCK DIAGRAM SHOWING RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NUMBER OF OCCURRENCES FOR A DISCRETE VALUE OF NATURAL FREQUENCY AND DAMPING PARAMETER.

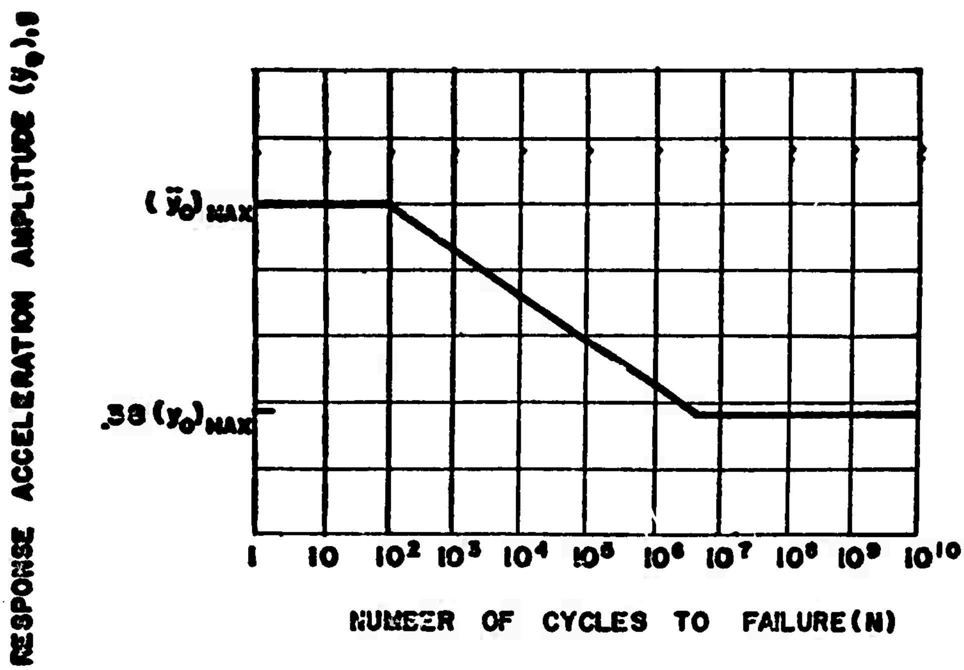


FIGURE 19. IDEALIZED CURVE OF RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NUMBER OF CYCLES TO FAILURE.

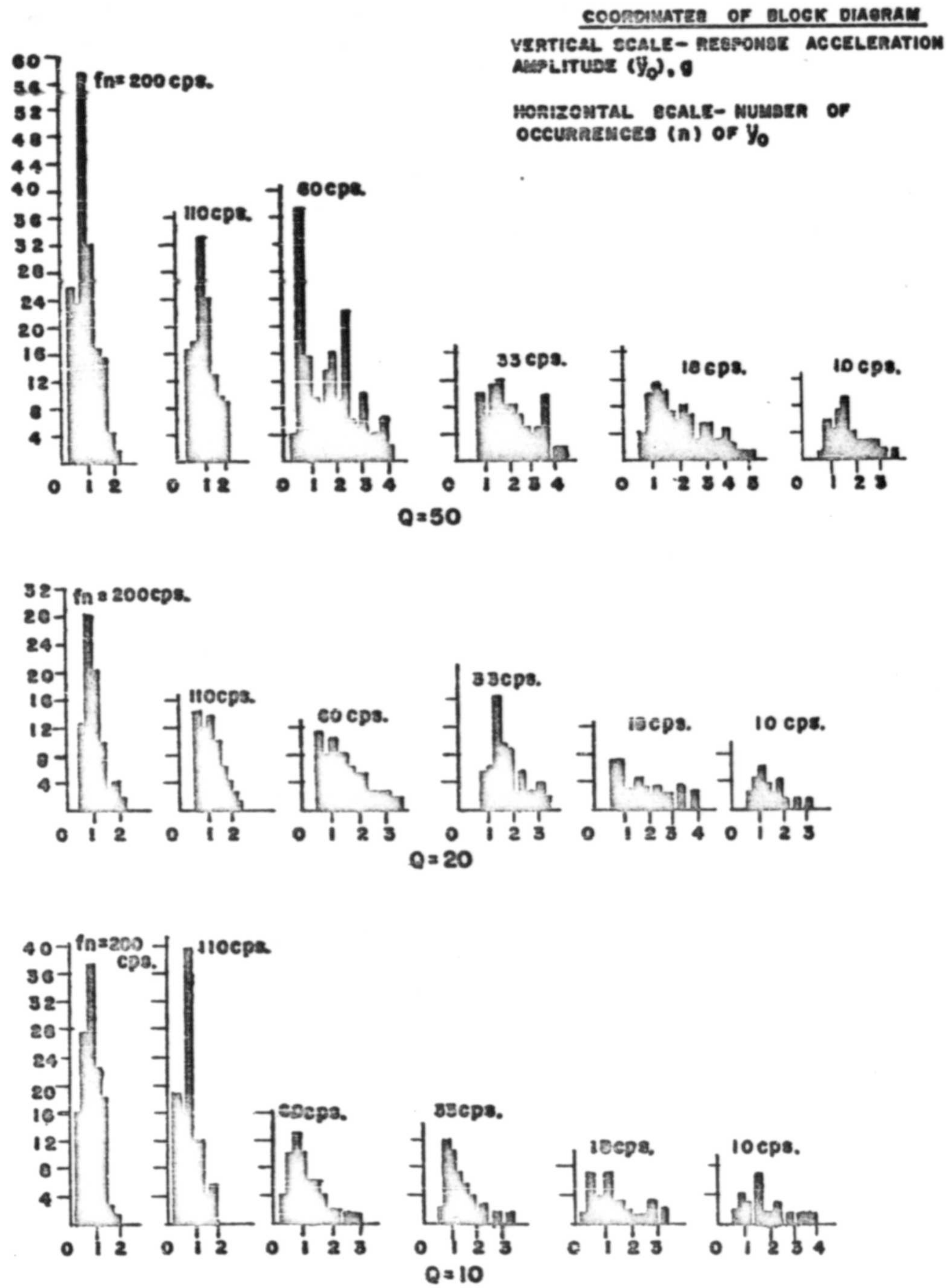


FIGURE 20· RESULTANT BLOCK DIAGRAMS FOR SIX LANDINGS SHOCKS AND SYSTEMS HAVING Q =10,20,50·

REQUIREMENTS FOR A LABORATORY SHOCK TEST

The problem of devising a laboratory shock test to simulate the most severe landing conditions, as defined by the resultant block diagrams in Figure 20, may now be approached. The block diagrams in Figure 20 represent the number of occurrences of response acceleration amplitude $\ddot{y}_o$ in one landing. An equivalent shock test would have a similar block diagram provided it were desirable to make the number of applications of shock in the laboratory test equal the number of landings experienced by the airplane during its life. In general, this is not a practical requirement inasmuch as the laboratory test should embody fewer applications of shock. Re-examining the Miner hypothesis for cumulative damage in fatigue and assuming that Figure 20 represents a single landing, equations (8) and (9) which refer to cumulative damage in fatigue may be written as follows to define conditions of failure:

$$D\left(\frac{n_1}{N_1}+\frac{n_2}{N_2}+\frac{n_3}{N_3}+\cdots\cdots\cdots\right)=1 \tag{10}$$

where D represents the number of landings in the life of the aircraft and n represents the number of occurrences per landing of each response acceleration amplitude, as indicated in Figure 20. Equation (10) is evaluated for each discrete value of natural frequency and damping ratio. Applying a similar analysis to block diagrams which can be obtained from laboratory records, the same structure may be considered to experience failure during the shock test when

$$D'\left(\frac{n_1'}{N_1'}+\frac{n_2'}{N_2'}+\frac{n_3'}{N_3'}+\cdots\cdots\cdots\right)=1 \tag{11}$$

where D' represents the number of repetitions of the laboratory shcok test and values of n' are obtained from block diagrams similar to Figure 20. These block diagrams are obtained by analyzing an oscillogram of acceleration as a function of time as measured on the shock testing machine, using the analog computer to determine the response acceleration amplitudes of simple systems as previously described.

To determine the relation between the resultant response surface for the landing shocks and the response surface for the laboratory shock, it is necessary to establish the relation between the values of N in equation (10) and the corresponding response acceleration amplitude. This was left undetermined when numerical values on the scale of response acceleration amplitude in Figure 19 were omitted. The required relation is now determined by evaluating equation (10) by a cut-and-try procedure to obtain a numerical value for $(\ddot{y}_o)$ max in Figure 19.

To evaluate equation (10), it is necessary that numerical values be available for parameters $\underline{D}$, $\underline{n}$ and $\underline{N}$. Values of $\underline{n}$ and $\ddot{Y}_o$ are known from the resultant block diagrams of Figure 20. The values of $\underline{N}$ cannot be determined unless numerical values are assigned to the ordinate scale; i.e., unless $(\ddot{Y}_o)$ max is known quantitatively. Numerical values are first assumed for D and for $(\ddot{Y}_o)$ max, thus making it possible to determine $\underline{N}$ from Figure 19. Using this value of $\underline{N}$ and the assumed value for $\underline{D}$, the left hand side of equation (10) is evaluated numerically. If the result is less than unity, a lower value is assumed for $(\ddot{Y}_o)$ max and the calculation repeated. If the result is greater than unity, a higher value is assumed for $(\ddot{Y}_o)$ max. This process is continued until the left hand side of equation (10) is made equal to unity.

The value assumed for the parameter $\underline{D}$ depends upon operating conditions. This parameter represents, for example, the number of landings that an aircraft experiences during its life. To investigate the effect that the value of $\underline{D}$ has upon the result, numerical values of 10,000 and 30,000 respectively, were assumed for $\underline{D}$. Equation (10) was then evaluated for each of these assumed values for $\underline{D}$, and values of $(\ddot{Y}_o)$ max as a function of frequency were calculated as shown in Figure 21. This result is obtained from the resultant block diagram of Figure 20 for a value of $\underline{Q}$ = 20. It is evident from Figure 21 that the resulting value of $(\ddot{Y}_o)$ max is substantially independent of the value assumed for the parameter $\underline{D}$ within a reasonable range. In view of this result, all subsequent calculations of this type assume a value $\underline{D}$ = 30,000 for the purpose of evaluating equation (10).

The curve of maximum response acceleration amplitude $(\ddot{Y}_o)$ max as a function of frequency, shown in Figure 21, is the intersection of any plane at $\underline{n}_t \leq 10^3$ with the minimum acceptable response surface for the shock being analyzed. By definition, $\underline{n}_t = \underline{Dn}$. The complete surface is shown in Figure 22. In creating this minimum acceptable response surface, it is assumed that the intersection of the surface with any plane parallel to the $\ddot{Y}_o - \underline{f}_n$ plane has a shape geometrically similar to Figure 21 but wherein the ordinates are a function of the particular value of $\underline{n}$. On the other hand, the intersection of the minimum acceptable response surface with any plane parallel to the $\ddot{Y}_o - \underline{n}_t$ plane consists of a straight, inclined line extending from $\underline{n}_t = 10^3$ to $\underline{n}_t = 5 \times 10^6$ and horizontally extending lines for smaller or larger values of $\underline{n}$.

The minimum acceptable response surface illustrated in Figure 22 is very important in the consideration of transient vibration or shock being developed here. This surface may be designated as a surface of required strength for general aircraft use. If an equipment cannot withstand the indicated response acceleration amplitude $\ddot{Y}_o$ for the number of cycles $\underline{n}_t$

set forth in Figure 22, it must be considered unsuitable for service. This condition must be met for any arbitrarily chosen value of n_t, and therefore may be used as a criterion to establish the validity of a laboratory test. A value of n_t applicable to laboratory testing may be defined as $n_t' = D'n'$. In this definition, D' is the number of repetitions of the laboratory test, and n is obtained from a series of block diagrams similar to Figure 20 but representing an analysis of an oscillogram obtained from the shock testing machine. By assuming a value of number of test repetitions D', a response surface $n_t' - \ddot{y}_o - f_n$ may be superimposed upon the response surface shown in Figure 22. This response surface for laboratory tests will generally involve relatively small values for n_t', and must lie above the surface for the environment shown in Figure 22.

In the preceding discussion, the maximum environment was obtained by inspecting the individual block diagrams and creating the resultant block diagrams of Figure 20. These resultant block diagrams represent the most severe environment, regardless of the source of shock, and were used to determine the curve of $(\ddot{y}_o)$ max shown in Figure 21. From this, the minimum acceptable response surface shown in Figure 22 was derived. An alternate approach to this problem involves the solution of equation (10) by cut-and-try methods for each individual landing shock, and the ultimate combining of the results to obtain an envelope of maximum values. The results of this analysis for six different landing shocksusing $D = 30,000$ are shown by the values of $(\ddot{y}_o)$ max set forth on Figure 23 to 25 inclusive, for values of $Q = 10$, 20 and 50, respectively. It is evident from an inspection of these figures that one landing shock may be more severe with respect to systems of a certain natural frequency, while another landing shock may be more severe with respect to systems of a certain natural frequency, while another landing shock may be more severe with respect to systems of other natural frequencies. An envelope encompassing the maximum response acceleration amplitudes at each natural frequency, regardless of landing shock, thus represents the most severe environment to be expected as a result of any landing. These envelopes are drawn in Figure 26 for values $Q = 10$, 20, and 50.

A comparison of the maximum response acceleration amplitude $(\ddot{y}_o)$ max is shown in Figure 27 for the alternate methods of calculation, using the value $Q = 20$ for the damping parameter. The solid line is reproduced from the curve for $D = 30,000$ in Figure 21, and is obtained by evaluating equation (10) directly from the resultant block diagram of Figure 20. The dotted line is the envelope of maximum response acceleration amplitude $(\ddot{y}_o)$ max when $Q = 20$, for all landings investigated individually as shown on Figure 26. The agreement between the results obtained by these alternate methods is good, and justifies the less laborious procedure which uses the composite block diagram of Figure 20. The curves shown in Figure 27 will be identified here by the designation "cumulative damage criterion"

to indicate that they include the effect of cumulative damage from many cycles of smaller stress. This is in contrast to the following analysis which neglects the cumulative effect and considers only the damaging effect of the cycles of stress having the greatest magnitude.

The inclusion of the effects of cumulative damage in the analysis becomes somewhat involved because of the need for evaluating equation (10) by cut-and-try methods. If only the effect of the cycle having greatest stress is considered, the responses may be shown as illustrated in Figures 28 to 30 where the ordinate is the maximum value of response acceleration $\ddot{Y}$ for each natural frequency, independent of number of occurrences. The designation "shock spectra" has received widespread acceptance to describe curves of the type set forth im Figures 28 to 30. Each of Figures 28 to 30 applies to a different value of $\underline{Q}$, and each line on a particular figure refers to a different landing shock.

To show a comparison between the shock spectra of Figures 28 to 30 and the curves which consider cumulative damage, the envelope representing the maximum value of response acceleration shown on Figure 29 for $\underline{Q}$ = 20 is reproduced as curve $\underline{A}$ in Figure 31. Inasmuch as $\underline{D}$ = 30,000 in the preceding calculation, it is assumed now that curve $\underline{A}$ represents the maximum value of response acceleration $\ddot{Y}$ that must be endured for 30,000 landings. In the previous calculation, values were calculated for $(\ddot{Y}_0)$ max on the basis that n_t = 1000 cycles of stress reversal. On the same basis, a curve corresponding to curve $\underline{A}$ but adjusted to 1,000 landings may be derived from curve $\underline{A}$ by referring to equation (7) and computing the ratio of accelerations $\ddot{Y}_0$ for a ratio of cycles $\underline{N}$ 30,000 to 1,000. This gives an acceleration ratio of 1.47. The ordinates of curve $\underline{A}$ in Figure 31 are now multiplied by 1.47 to obtain curve $\underline{B}$. This curve represents the required response acceleration $\ddot{Y}$ assuming 1000 cycles of stress reversal, and may thus be compared with the results previously obtained considering the effect of cumulative damage. To facilitate this comparison, the dotted curve on Figure 27 is reproduced as curve $\underline{C}$ in Figure 31. The extent of the agreement between curves $\underline{B}$ and $\underline{C}$ in Figure 31 is an indication of the effect of cumulative damage in evaluating the damaging potential of repeated shocks. The spacing between these curves suggests that the cumulative effect of cycles of lower stress is appreciable but not so great as to warrant the additional analytical time necessary to make the correction. This should not be construed as a generalization but only a tentative opinion on present conditions. A more complete investigation of this and other problems may lead to a different conclusion.

The minimum acceptable response surface shown in Figure 22 represents the required strength of any equipment which will be subjected to the landing shocks being analyzed here. It may be determined by any convenient analysis, as pointed out in the

preceding discussion. Since one of the parameters of the surface is total number of stress reversals experienced by the equipment during its life, it is possible to devise a transformation from actual environment to laboratory conditions. The latter necessarily involves fewer cycles of stress reversal. An analysis based on laboratory testing conditions, however, must yield values of $\underline{Y_o}$ falling above the minimum acceptable response surface for appropriate values of $\underline{f_n}$ and $\underline{n_t}$.

Following the hypothesis formulated by Miner to account for the effect of cumulative damage in fatigue, a shock test is suitable for qualifying equipment to withstand the shock represented by the minimum acceptable response surface shown in Figure 22 if it meets the following requirement:

$$D'\left(\Sigma \frac{n'}{N}\right) = 1 \tag{12}$$

where $\underline{D'}$ represents the number of applications of shock during the test, $\underline{n'}$ represents the number of occurrences of various maximum values of acceleration response, and $\underline{N}$ is taken from Figure 19. Equation (12) is patterned after equation (10), and must be applied at each of several natural frequencies. Considerable discernment is necessary in establishing a suitable level for the shock test. It is important that $\underline{D'}$, the number of applications of shock, not be too great because the test then tends to become laborious. On the other hand, if $\underline{D'}$ is made too small, the required response acceleration tends to become unduly great and the assumption of linearity for the structure becomes invalid. In other words, referring to Figure 19, the part of the curve for $N < 10^3$ becomes applicable. The need for maintaining $\underline{D'}$ relatively great perhaps suggests that the shock testing machine should be of an automatically repeating type.

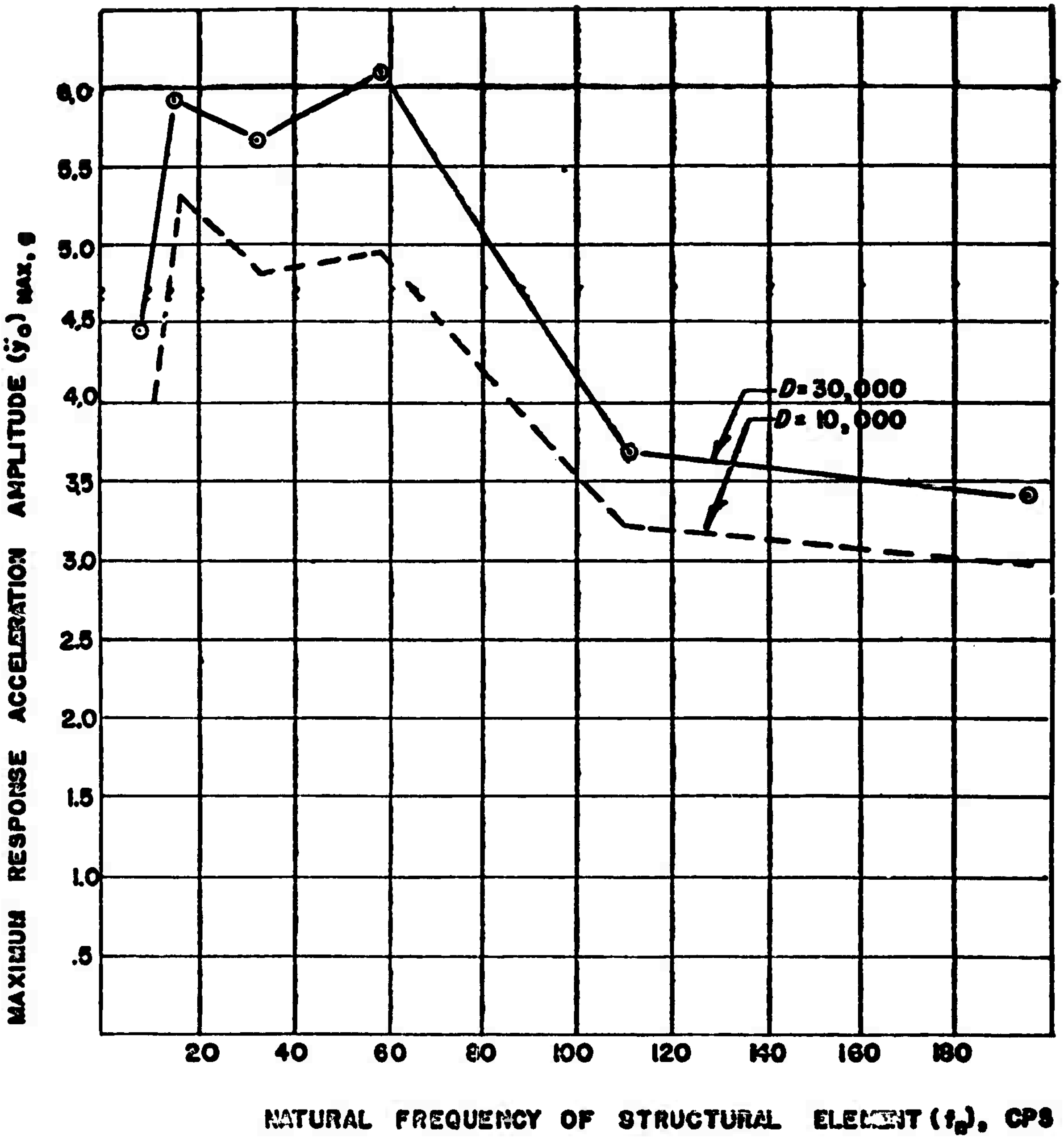


FIGURE 21. MAXIMUM RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NATURAL FREQUENCY FOR ASSUMED VALUES D = 10,000 AND 30,000. DAMPING PARAMETER $\theta = 20$. THESE CURVES ARE INTERSECTION OF MINIMUM ACCEPTABLE RESPONSE SURFACE WITH PLANE $n_t = 10^3$ WHERE $n_t = D\,a$.

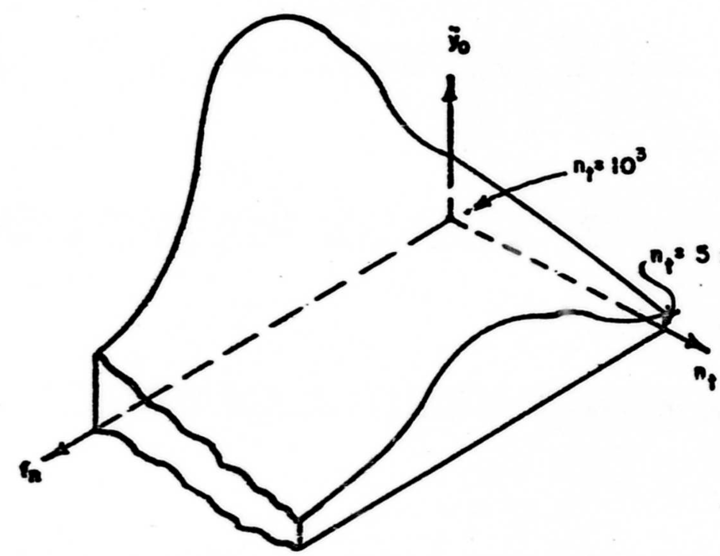


FIGURE 22. ILLUSTRATION OF TYPICAL MINIMUM ACCEPTABLE RESPONSE SURFACE.

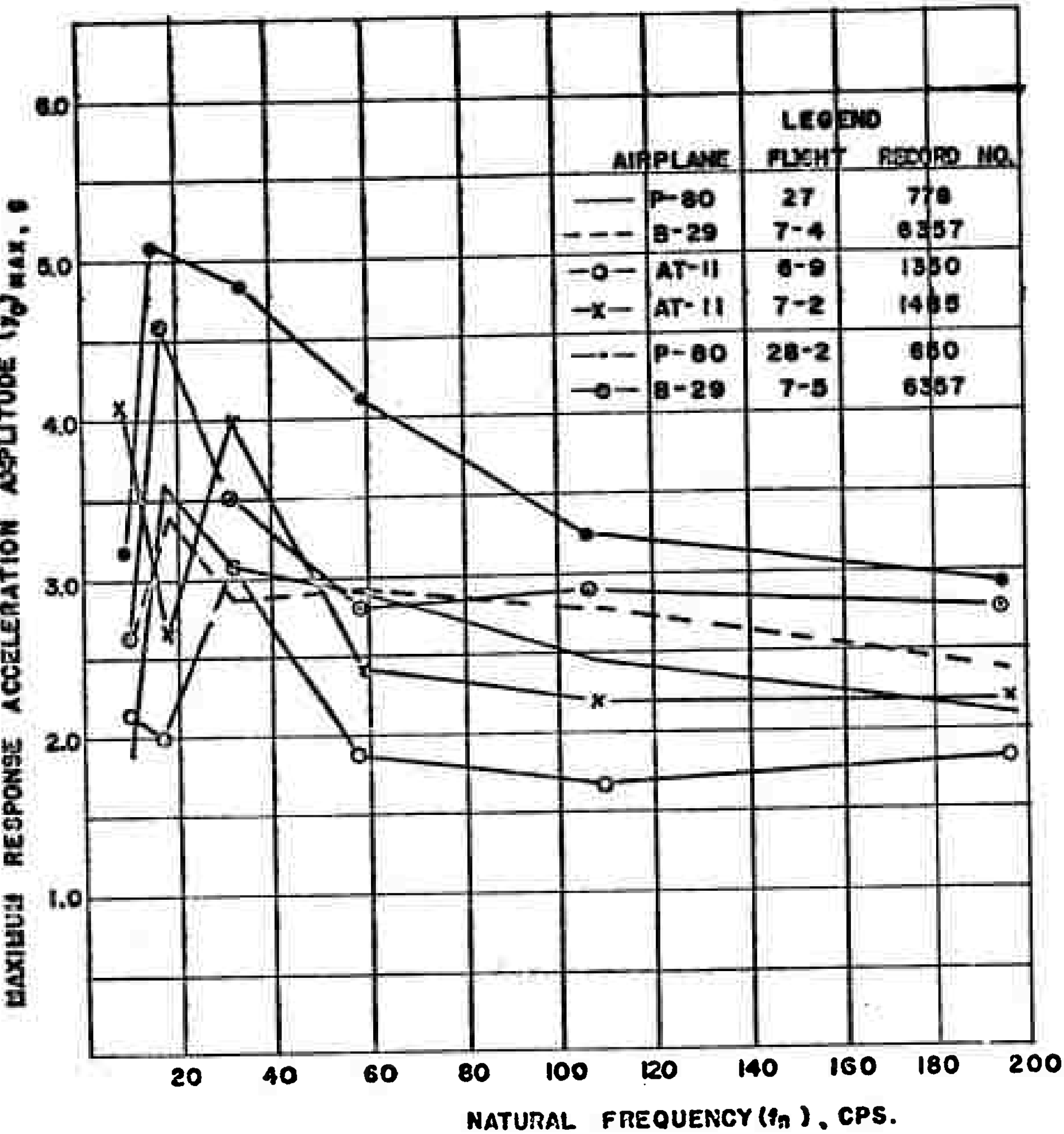


FIGURE 23. CURVES OF MAXIMUM RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NATURAL FREQUENCY FOR INDIVIDUAL LANDING SHOCKS WHEN Q = 10 AND D = 30,000

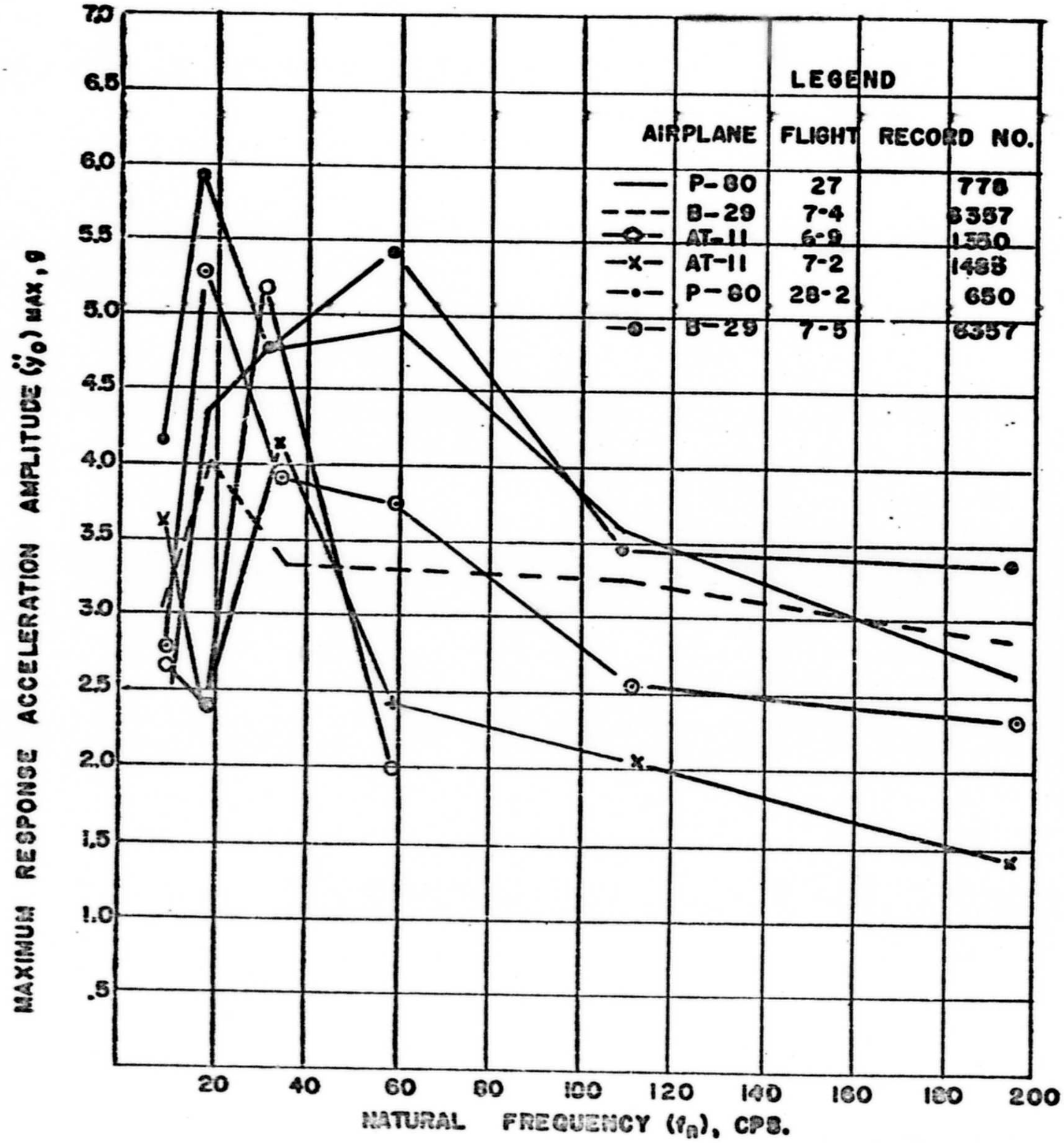


FIGURE 24. CURVES OF MAXIMUM RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NATURAL FREQUENCY FOR INDIVIDUAL LANDING SHOCKS WHEN θ = 20 AND D = 30,000

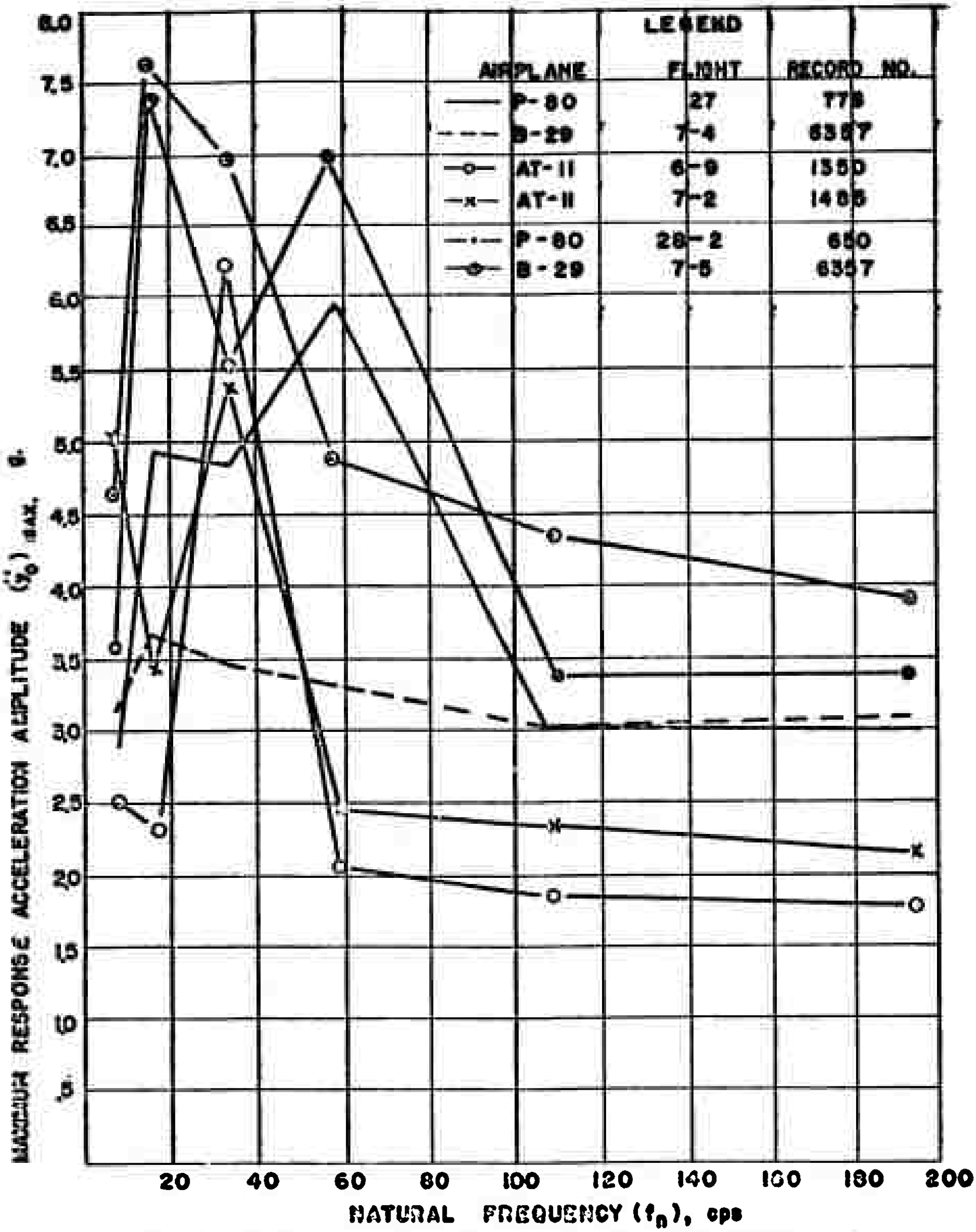


FIGURE 25. CURVES OF MAXIMUM RESPONSE ACCELERATION AS A FUNCTION OF NATURAL FREQUENCY FOR INDIVIDUAL LANDING SHOCKS WHEN $Q = 50$ AND $D = 30{,}000$.

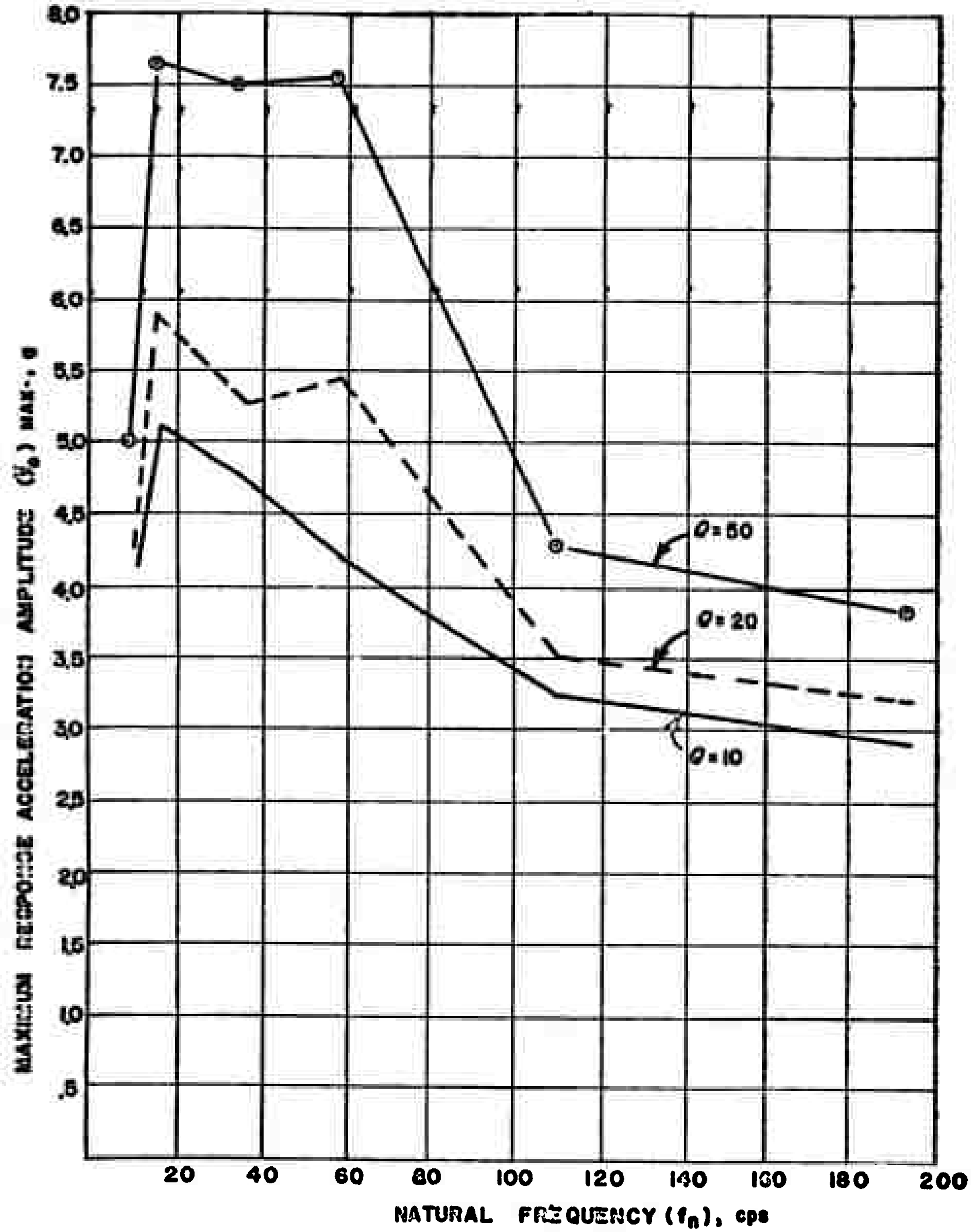


FIGURE 26. ENVELOPES OF CURVES OF ($\ddot{y}_o$) MAXIMUM IN FIGURES 23 TO 25, FOR VALUES Q = 10, 20 AND 50.

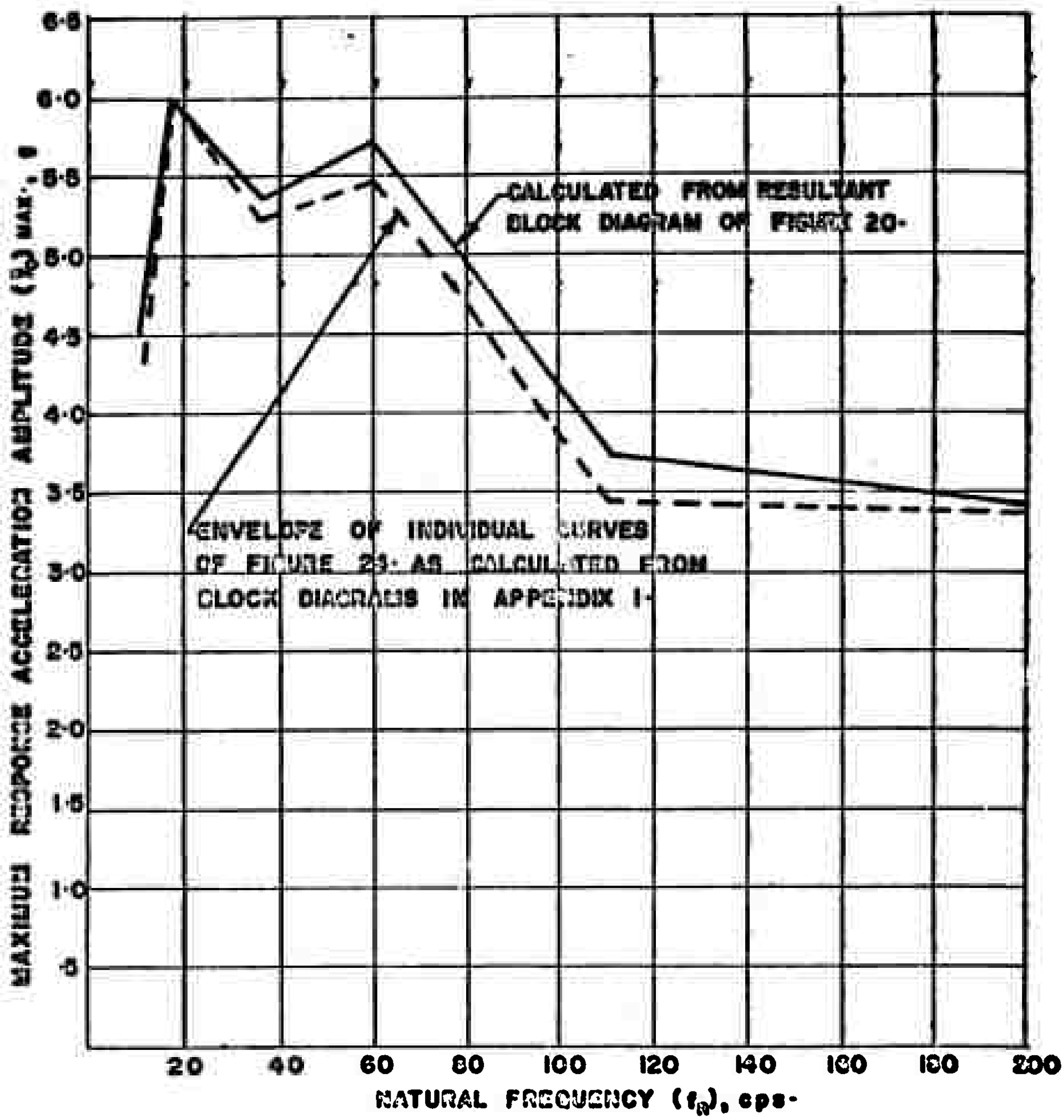


FIGURE 27. MAXIMUM RESPONSE ACCELERATION AMPLITUDE AS A FUNCTION OF NATURAL FREQUENCY FOR $Q = 20$, SHOWING COMPARISON OF RESULTS OBTAINED BY ALTERNATE METHODS OF CALCULATION.

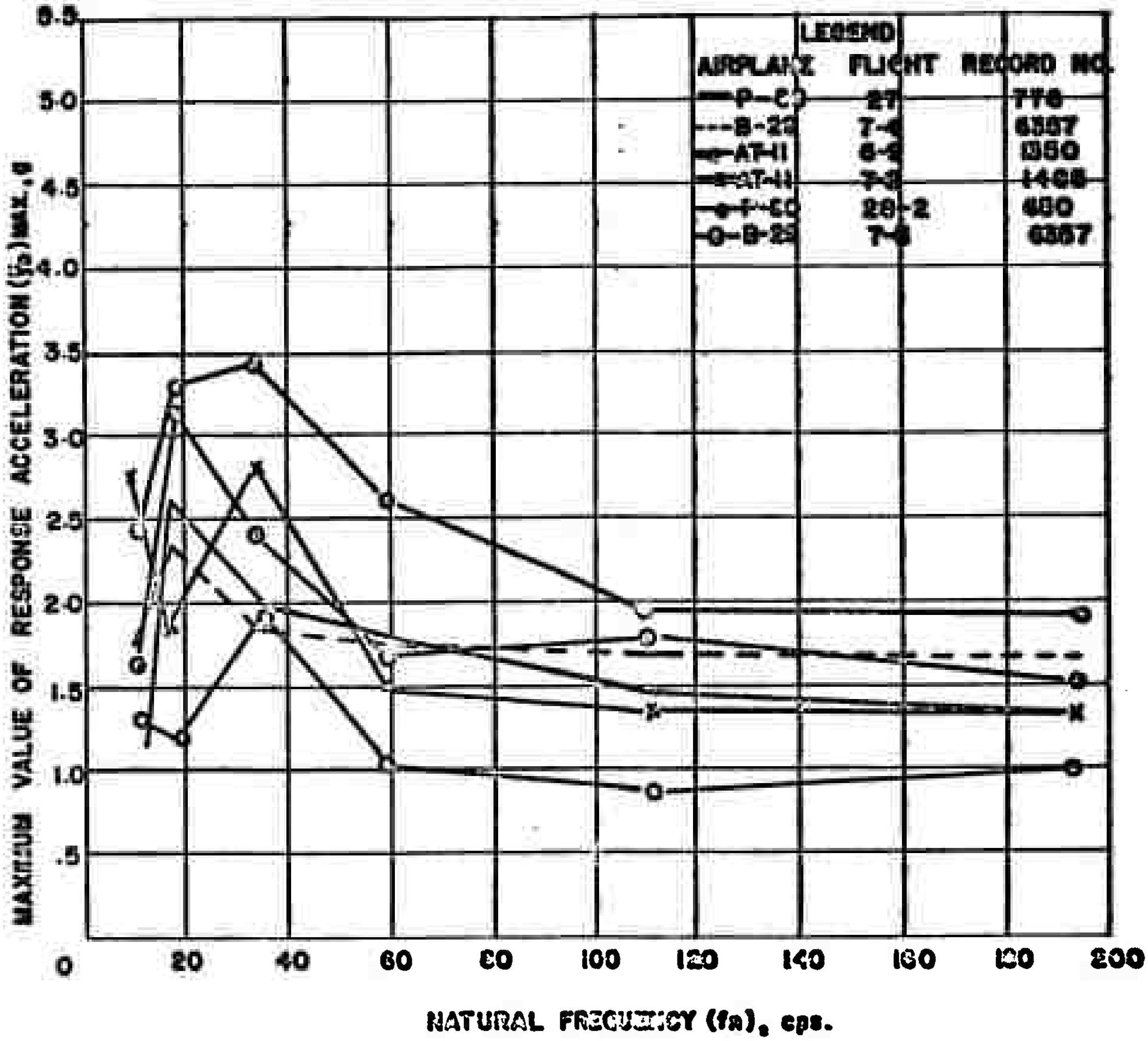


FIGURE 20. SHOCK SPECTRA FOR INDIVIDUAL LANDING RECORDS, θ = 10.

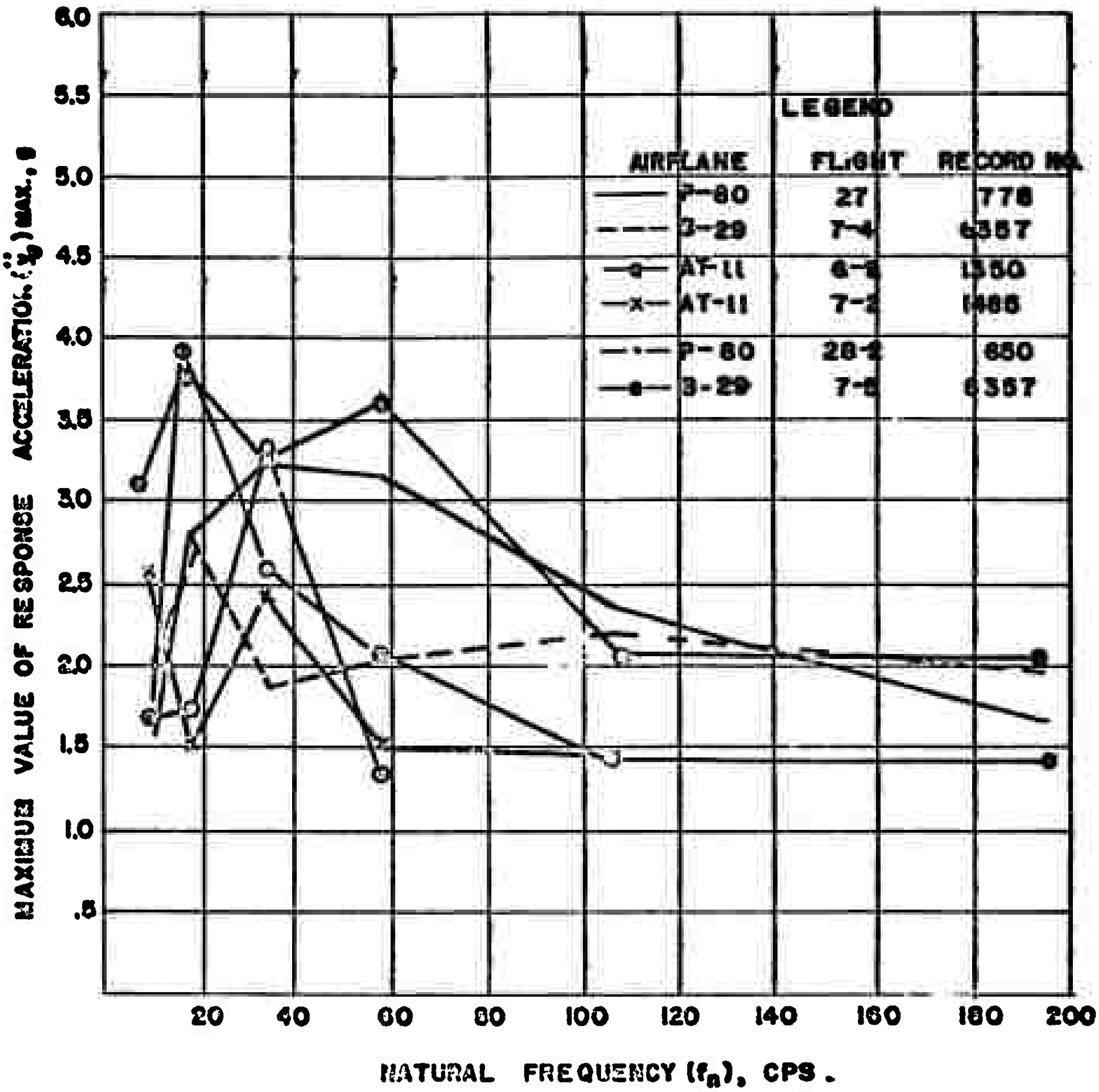


FIGURE 29. SHOCK SPECTRA FOR INDIVIDUAL LANDING RECORDS, $Q = 20$.

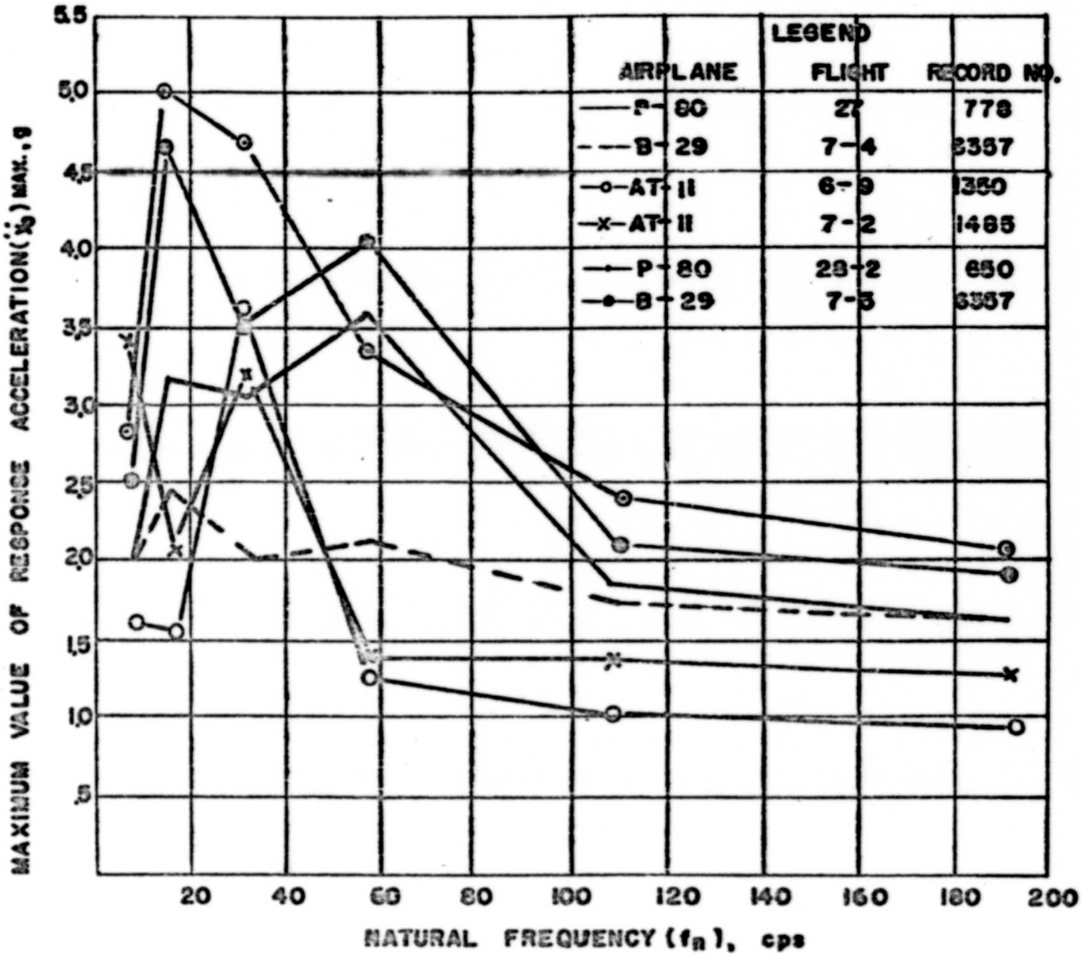


FIGURE 30. SHOCK SPECTRA FOR INDIVIDUAL LANDING RECORDS, $Q = 30$.

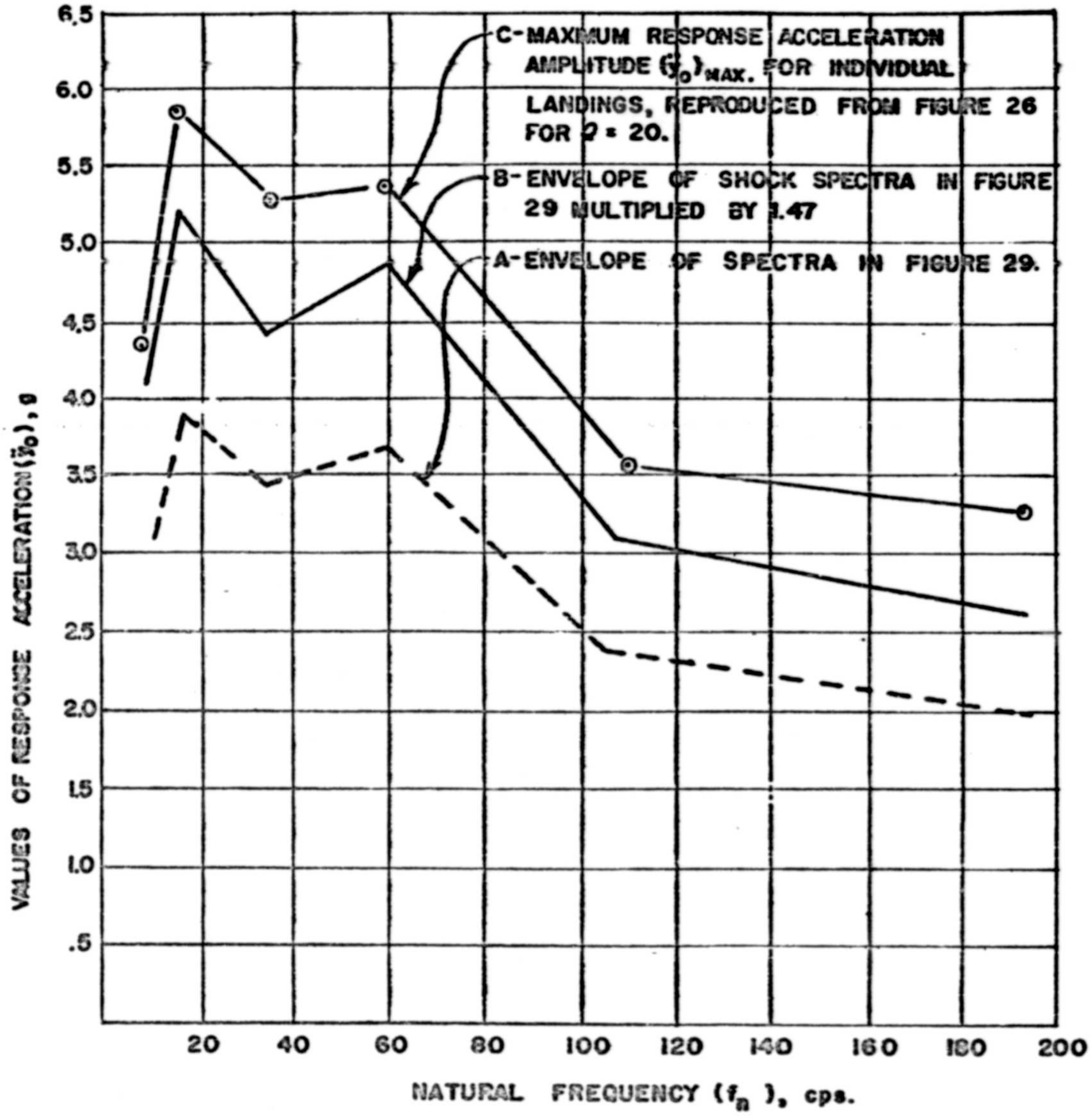


FIGURE 31. COMPARISON OF STRENGTH REQUIREMENTS AS CALCULATED FROM SHOCK SPECTRA AND CUMULATIVE DAMAGE THEORY.

SHOCK TESTING PROCEDURES

Several different types of shock testing machines are in current use for testing equipment intended for airborne service. One of the most commonly used machines is the Type 150-400 VD Variable Duration Shock Testing Machine. The equipment under test is attached to an elevator constrained by suitable guides arranged to move only vertically. To conduct shock tests, the elevator is lifted by a cable arrangement a pre-determined distance by suitable means, and is permitted to fall freely. Downward motion of the elevator is arrested as it falls into a sand box constituting the lower part of the shock testing machine. The bottom of the elevator carries an array of wooden cleats whose arrangement may be varied to change the suddenness of application of the decelerating force. Current specifications often call for shock tests of two different degrees of severity. One of these involves a free fall of approximately four inches, while the other involves a free fall of 13 inches. The time histories of acceleration as measured on the elevator are shown by the oscillograms set forth in Figures 32 (A) and (B), respectively.

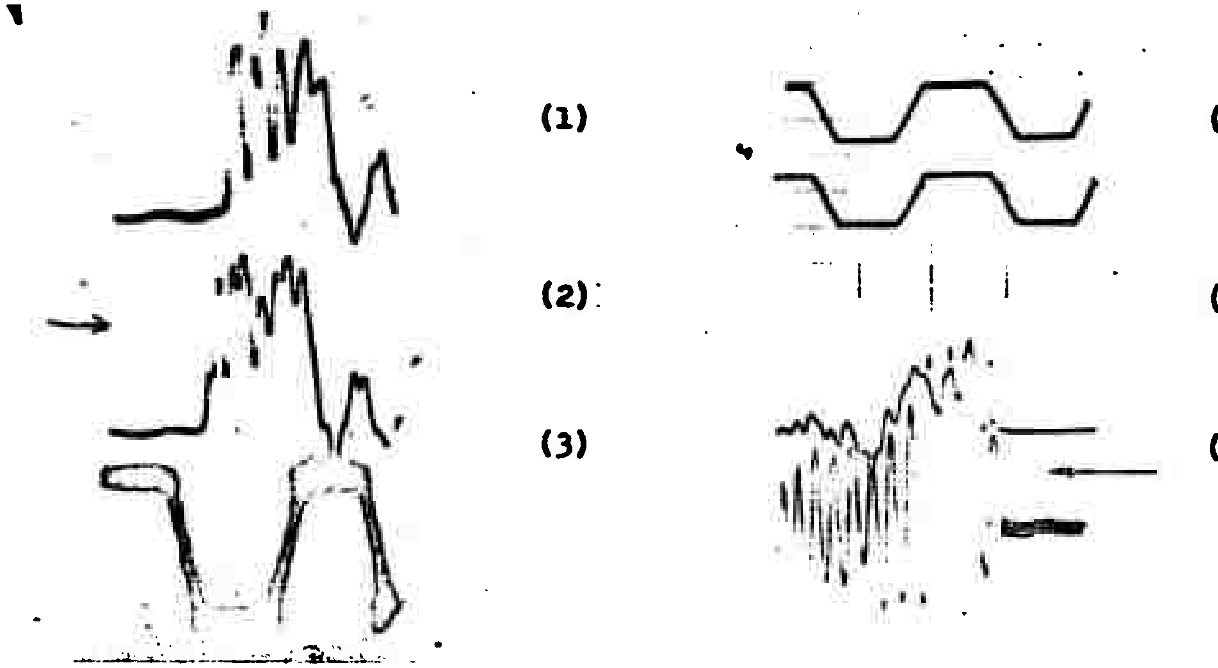
The oscillograms shown in Figure 32 have been fed into the analog computer, and shock spectra have been obtained. These shock spectra take into consideration only the maximum response acceleration; they are shown graphically in Figure 33 for both the 4 inch and 13 inch free fall. These shock spectra show excessively high values at natural frequencies of approximately 500 cps, the frequency of the predominent vibration which is superimposed upon the pulses shown in Figure 32. This superimposed vibration excites a structure having a natural frequency of 500 cps and causes a pronounced peak in the shock spectrum. This is in contrast to the records obtained from measurements made in service conditions where the lack of high frequencies in the record suggests limitations in the instruments.

To afford a comparison of the shock machine output and the shock experienced during landing, as determined from the preceding analysis, curve B is reproduced on Figure 33 from Figure 31. This curve includes a factor, noted in connection with Figure 31, which transforms the magnitude of the response acceleration to a value which is appropriate for 1000 cycles of stress reversal. Current shock testing procedures involve substantially fewer than 1000 cycles of stress reversal. Consequently, the lower curve in Figure 33 is not strictly comparable with the upper curve because of this disparity in number of cycles. The curve representing the shock spectra of the landing shocks cannot be further transformed by an additional reduction in the number of cycles because of a discontinuity at N = 1000 cycles, as indicated in Figure 19.

The other alternative involves an increase in the number of applications of shock involved in a representative shock test. It will be one of the recommendations in this report that consideration be given to so modifying the shock testing procedure.

The curve describing the shock spectrum for individual landings in Figure 33 should substantially coincide with the shock spectrum for the four inch free fall of the shock testing machine, if the number of cycles of stress reversal were the same. The spread between the two curves appears too great to be accounted for entirely by number of cycles of stress reversal. This suggests that the laboratory shock testing requirements should be modified to call for (1) a significantly larger number of amplications of shock and (2) a substantially less severe shock. The latter conclusion must be qualified at this time because there is considerable eivdence that records defining landing shock of maximum expected severity were not made available to Contractor and have not been included in this analysis. If such records are made available during a supplemental program, an analysis of the records may reveal that the current shock testing procedure is valid insofar as severity is concerned. It appears desirable in any event to consider increasing the number of applications of shock in a typical test. Any changes in the shock testing program, together with possible changes in the design of shock testing equipment, will be considered in a supplemental program.

The shock spectrum representing the performance of the shock testing machine at a free fall of 13 inches, as shown by the upper curve in Figure 33, involves values of response acceleration far in excess of those indicated by the environmental conditions studied in this analysis. It is understood that the test involving a 13 inch free fall is intended to simulate conditions encountered during minor crashes; it would not be expected, therefore, that oscillograms obtained during normal landings would conform to crash conditions. It is understood by the authors of this report that the level of severity embodied in a 13 inch free fall was established by representatives of the United States Air Force during a study of several airplane crashes. Under these circumstances, and in the absence of numerical test data obtained during crashes, the authors feel that the shock test involving a 13 inch free fall should not be considered subject to review at this time. It is recommended, however, that every opportunity be used to re-examine the requirements for the more severe shock test with the objectives of determining whether the presently specified test is realistic and whether the test serves its primary function of insuring that equipment does not become a missile within personnel spaces during crash conditions.



(A) Four inch free fall
Time

(1) Acceleration on elevator
(2) Similar to (1)
(3) Calibration trace - 15g (peak-to-peak) and 60 cps.

(B) 13 inch free fall
Time

(1) Calibration traces - 30g (peak-to-peak) and 60 cps.
(2) Acceleration on elevator
(3) Strain in cantilever beam with natural frequency of 500 cps. mounted on elevator.

Figure 32. Oscillograms showing performance of 150-400 VD Medium Impact Shock Machine.

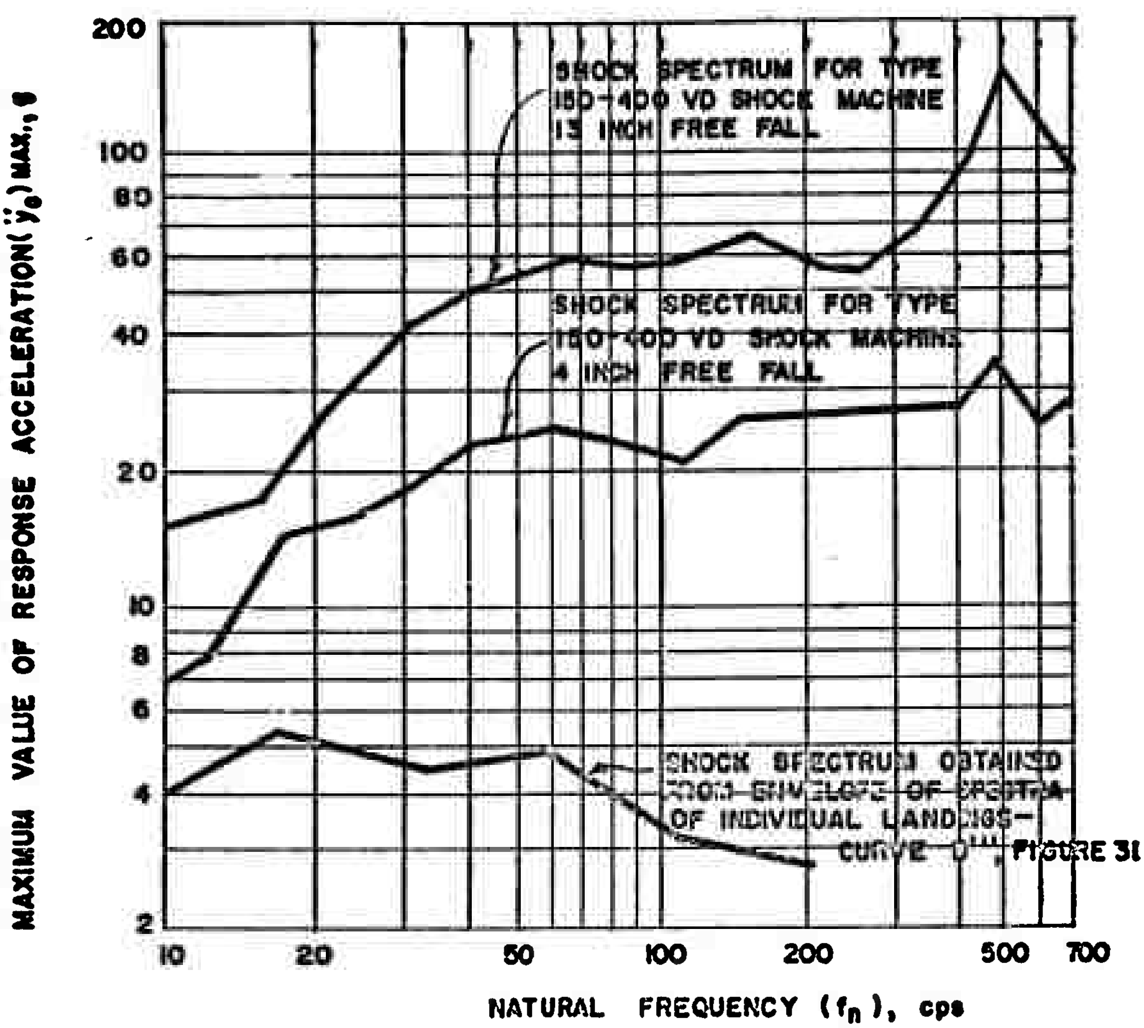


FIGURE 33. COMPARISON OF SHOCK SPECTRA FOR 150-400 VD. SHOCK MACHINE WITH SHOCK SPECTRA OBTAINED FROM ENVELOPE OF INDIVIDUAL LANDINGS.

OTHER CONSIDERATIONS

Use of Isolators

The analysis used in this investigation for both steady-state and transient conditions is based upon the assumption that the equipments being investigated can be idealized as single-degree-of-freedom systems. This is indicated in Figure 1, and the discussion pertaining thereto. Although the natural frequencies of structures in higher modes of vibration are of appreciable significance, it is felt that considerable insight into the strength of equipment subjected to vibration and shock can be achieved by considering the equipments and components thereof as simple systems. When such equipments are supported by isolators, an additional factor is introduced. Considering the equipment to be comprised of concentrated masses and massless springs, the equipment on isolators must be considered as at least a two-degree-of-freedom system. One of the degrees of freedom is associated with the entire equipment on isolators, while the other degree-of-freedom pertains to the individual components with reference to the chassis of the equipment. A doubt may arise concerning the validity of the results obtained on the basis of the single-degree-of-freedom concept. This question can best be answered by considering the steady-state and transient conditions independently.

The laboratory tests which simulate the conditions of steady-state vibration in aircraft are true simulated tests in the sense that the laboratory tests reproduce both the frequency and amplitude of the environment with such modifications as required to compensate for the relatively short duration of the laboratory tests. Isolators are frequency-sensitive elements, with the result that all amplitudes in a system retain their proportionality at a given frequency. In other words, the responses of components of the equipment have a certain relation to the environment independent of the vibration amplitude embodied in the environment. The use of isolators, therefore, does not influence the selection of the exaggeration factor employed to select an amplitude for the vibration test which simulates the steady-state aspect of the environment. The analysis which establishes the test for steady-state conditions thus remains rigorous, with or without the use of isolators to support the equipment during tests.

In the transient vibration or shock aspect of the problem, the laboratory test is not a direct simulation in the sense that frequencies and amplitudes are reproduced. The laboratory test is considered to simulate the environment if the responses of a wide range of equivalent systems are equivalent. Under these circumstances, it is probable but not essential that a large response in a system of a particular natural frequency will be the result of vibration present in the environment at this frequency. Inasmuch as isolators are frequency-responsive elements, they thus tend to function

during laboratory testing in a manner similar to that in which they function during actual service. A discrepancy in this reasoning may arise as a result of non-linearity in the isolators, particularly if the non-linearity is severe as in the case of hard bottoming. This introduces a degree of doubt regarding the validity of the test if isolators are used during the simulation of transient vibration or shock.

The above discussion indicates that the use of isolators during steady-state vibration does not invalidate the laboratory test. With regard to the shock tests used to simulate transient conditions, there is a point of doubt introduced primarily by the non-linearity of isolators. It is believed, however, that the conditions will be made more nearly representative of actual conditions by employing isolators than by eliminating them. As a consequence, it is recommended that any equipment mounted upon isolators in actual service installations be similarly mounted during laboratory tests.

Transportation and Handling

The ability of equipment to withstand the vibration and shock encountered during transportation by common carrier or otherwise is related to the problem of isolators because such equipment is normally protected by cushioning of some type during shipment. Although transportation involves the identical items of equipment considered in this report, the problem is basically unrelated to the investigation of environmental conditions in aircraft. The problems encountered and the strength required of the items being shipped are identical for the shipment of any commodity. It is understood that specifications and standard procedures have been established for the shipment of military supplies, and that the degree of success experienced during such shipment is being constantly monitored. It is evident that the nature of the vibration and shock encountered during shipment cannot be modified by this analysis. If the equipment is unable to withstand the treatment received during shipment, it should be redesigned or provided with better protection. With regard to protection of equipment during transportation, it is suggested that a study be made of the damage sustained by such equipment during transportation to serve as a guide to proper remedial action.

The term "ground handling shock" is sometimes used to designate the type of shock experienced by equipment as a result of handling during servicing, installation, and re-arrangement in aircraft during tactical use. It is understood that equipment is sometimes set down upon a hard bench or the floor, or upon a table of an aircraft where vibration is normally present. Under these types of handling, isolators often are not present to afford protection to the equipment. While it is conceded that the shock experienced during such handling is of importance, it does not appear possible to include a consideration of these conditions in this analysis for at least two reasons as follows:

(a) This analysis is based primarily upon numerical data representative of service conditions. From the nature of the circumstances, it does not appear feasible to collect numerical data representative of ground handling conditions.

(b) The avoidance of damage during ground handling conditions would appear to be a matter of education. Equipment is designed primarily for combat use in aircraft, and the tests recommended in this report are intended to insure that the equipment is suitable for its intended use. If the equipment is satisfactory for combat but subject to damage by handling, it is recommended that the persons responsible for such handling be educated in proper handling techniques.

CONCLUSIONS

The desired end result of the analysis reported here is a testing procedure for laboratory use, of established validity for certifying that electronic and accessory equipment is qualified to withstand the vibration and shock encountered in aircraft service. Inasmuch as the present program does not include within its scope the actual measurement of vibration and shock in aircraft, reliance must be placed on the use of existing measured data. Unfortunately, the data made available to the Contractor are incomplete in many instances, are of doubtful validity in other instances, and often are not presented in a form that is useful in the type of analysis carried out here. For these reasons, the conclusions set forth here must be regarded as tentative, pending the opportunity to remove the inadequacies in the data.

Although vibration and shock are often referred to as a type of environment, they are basically different phenomena. The former is steady-state in nature, while the latter is transient. Consequently, they require different methods of analysis, the results are presented in different forms, and different testing methods are required. They are considered separately throughout the report, and the conclusions are presented separately.

It is generally conceded that vibration tests should cover the frequency range between the lowest and highest vibration frequency encountered in the actual environment. The range of frequencies may be covered by continuously changing the test frequency in a pattern leading from minimum to maximum and back to minimum. As an alternative, the vibration test may be conducted at discrete frequencies separated by predetermined frequency intervals. For reasons which are set forth in the body of this report, it is concluded that the sweep frequency method is preferable. Recommendations with regard to the vibration test are as follows:

(a) It should be required that the equipment operate properly while vibrating at any frequency between 5 and 50 cycles per second with a displacement amplitude of 0.030 inch peak-to-peak, and at any frequency between 50 and 500 cycles per second with an acceleration amplitude of $\pm 4g$. These values represent the Contractor's estimate of expected maximum environments. This test is only to insure that the equipment will remain operative when subjected to the actual environment.

(b) The equipment should be required to withstand, without failure, vibration at a displacement amplitude of 0.070 inch peak-to-peak at all frequencies between 5 and 50 cycles per second, and an acceleration amplitude of ±9.5g at all frequencies between 50 and 500 cycles per second. This test is to be continued for a period of approximately four hours in each of three directions, and involves the rates of change of frequency set forth in Figure 13a of the body of this report. This is the accelerated vibration test. It is not contemplated that equipment be required to operate properly during this test, but that it should be operative at the conclusion of the test and that it should not sustain damage during the test.

Transient vibration or shock cannot be defined in terms of frequency and amplitude. Consequently, the preceding type of analysis is inapplicable to a consideration of transient conditions. A time history of the transient environment is required. A group of oscillograms giving the time history of acceleration experienced during landing of aircraft was made available to the Contractor. From among this group of records, certain records were selected for analysis. Insofar as could be determined prior to analysis, these records were selected on the basis that they should be the most severe of the group, and represent a diversity of characteristics so that the result would be representative of a range of transient conditions. There is reason to believe these records do not approach the level of severity that may be embodied in records available from other sources. For this reason, the analysis cannot be considered complete at this time, and no conclusion has been reached finally. As a result of the analysis completed at this time, however, certain tentative opinions may be expressed regarding the shock test:

(a) The analysis does not indicate a level of shock as great as that embodied in the conventional laboratory test involving a height of drop of 13 inches, often referred to as a test with a maximum acceleration of 30g and a duration of 0.011 second. This conclusion apparently is justified, inasmuch as the 13 inch shock test is considered to be representative of crash conditions, and the records available to the Contractor do not include crash conditions.

(b) Current shock testing specifications also call for a shock test involving a free fall of 4 inches, often referred to as a test with a maximum acceleration of 15g and a duration of 0.011 second. This is considered to be representative of severe operating conditions. The analysis carried out here indicates that the shock test should be modified to include a substantially greater number of applications of shock, perhaps at a somewhat lower severity. The desired level of the test is established only tentatively because it is suspected that available oscillograms do not show maximum expected severity.

The analysis of steady-state vibration included in this report is predicated primarily upon the data set forth in Reference 55. As explained in a preceding paragraph, an attempt was made to re-evaluate the data with the objective of eliminating non-representative portions thereof. This re-evaluation was of a cursory nature, however, because it was not included within the scope of the project as initially established, and resources were not available to make a comprehensive re-examination. The authors remain somewhat dissatisfied with the result, and suggest a comprehensive re-evaluation of steady-state vibration conditions to include the following:

(a) It should be determined by examining representative portions of the original records that the reported amplitudes and frequencies are sufficiently periodic to make a tabular presentation significant. There is reason to suspect that certain data in Reference 55 which appear to indicate steady-state conditions actually refer to transient conditions.

(b) Where amplitudes and frequencies are reported in numerical terms, it should be a requirement of such a presentation that a harmonic analysis of the original oscillogram has been made. Attempts to determine amplitude and frequency of a complex record by inspection may be misleading unless care is exercised.

(c) Much of the data set forth in Reference 55 refer to measurements made at locations within the aircraft not representative of positions for installation of electronic or accessory equipment. Many points can be eliminated from the tabular data in Reference 55 by noting the location of the measurement. The effects of such elimination upon the graphical presentation cannot be determined, however, without entirely replotting the graphs. It is questionable that a mere replot of the tabular data is justified without at the same time examining such data to determine that they represent steady-

envelope would be experienced or duplicated a given number of times in the life of the aircraft appears to be the only logical assumption in the absence of statistical data indicating the probable occurrences of various levels of shock severity.

Oscillograms defining conditions of transient vibration or shock which were made available to the Contractor do not indicate the severity believed to occur in certain phases of aircraft operation. This part of the analysis was of necessity carried out last because of delays encountered in placing the analog computer into operation. Consequently, time was not available to obtain and analyze additional records showing shock of maximum severity. It is hoped that this deficiency can be removed in a supplemental program, and that agencies having cognizance of operations in which shock of greatest severity occurs will cooperate by making records of such shock available for analysis. In the meantime, conclusions reached in this report with respect to shock tests must be considered tentative and probably not severe enough.

It is pointed out in this report that a transient vibration or shock can best be evaluated by determining the response of systems subjected to such vibration or shock. This can be done quite readily for single-degree-of-freedom systems having natural frequencies and damping parameters within representative ranges. Many actual equipments involve not single-degree-of-freedom systems but multi-degree-of-freedom systems; non-linear elasticity often exists in place of the assumed linear elasticity. No attempt was made in this analysis to investigate the effect of either added degrees of freedom or non-linearity. While it is believed by the authors that a damped, linear, single-degree-of-freedom system may serve as a fairly satisfactory criterion for comparing environmental conditions with laboratory test conditions, it seems evident that future investigations should include the effects of non-linearity and additional degrees of freedom if the method of analysis is to be refined.

This report includes data on the response acceleration which serves to define the environmental conditions arising from shock or transient vibration. Methods of using such response data to formulate a requirement for laboratory testing are explained. A brief analysis of current shock testing machines is included, and it is indicated that certain modifications in the shock testing machines and testing procedures may be desirable. This report does not include within its scope a study of the changes in testing machine design required to achieve the desired correspondence between environmental and laboratory conditions. It is recommended that the program be continued to make the necessary modifications in laboratory testing equipment and testing procedure.

A program should be established to validate the conclusions reached here on the basis of hypothesis and idealization. Such validation can be obtained only by comparing the results of laboratory tests of equipment with the experience gained by operation of such equipment in actual flight environments. It is recommended that a system be established for comparing these experiences. Failure of any equipment, with as much detail on failed components as possible, should be reported to a central agency whenever the failure appears to be the result of vibration or shock. If similar failures are not noted as a result of laboratory testing, there would be an indication that the tests are not effective in simulating the environmental conditions encountered in flight. This would then call for a re-evaluation of laboratory testing conditions. It is recommended that such a data collection procedure be instituted immediately, and that the system include means to compare the results of laboratory testing with actual experience in flight environments.

The ability of equipment to withstand transient environmental conditions is a function not only of the environment but also of the strength of the equipment. A particular equipment may be constructed with relatively great strength and, at the same time, be particularly vulnerable to a given environment if the environment includes vibration at frequencies which correspond to the natural frequencies of critical components of the equipment. On the other hand, another equipment with much lower inherent strength may be able to better withstand the same environment because this coincidence of frequencies does not exist. Conclusions reached in this analysis are based upon an idealization of an equipment as a single-degree-of-freedom system. This type of idealization undoubtedly overlooks many important factors which can be included only by extending the analysis to include various types of multi-degree-of-freedom systems. This would make it possible to include the effect of relatively heavy components overlooked in the current analysis, and to better evaluate the effect of isolators. This latter evaluation would be particularly effective if it could include non-linearity in the characteristics of isolators.

APPENDIX I

BLOCK DIAGRAMS OF RESPONSE ACCELERATION

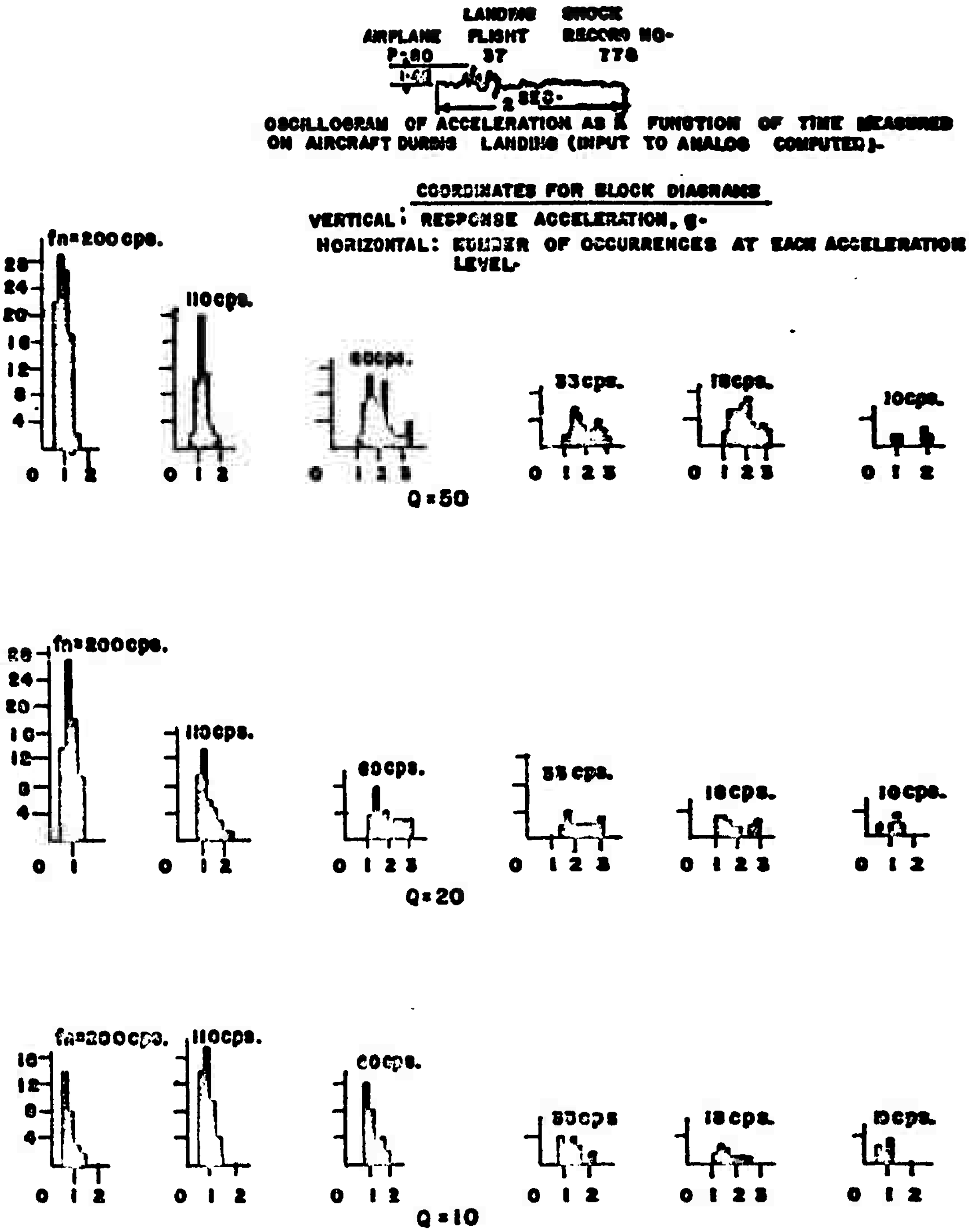


FIGURE X-1. BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF NUMBER OF OCCURRENCES FOR SYSTEMS HAVING Q=10, 20, & 50, SUBJECTED TO P-80 LANDING SHOCK.

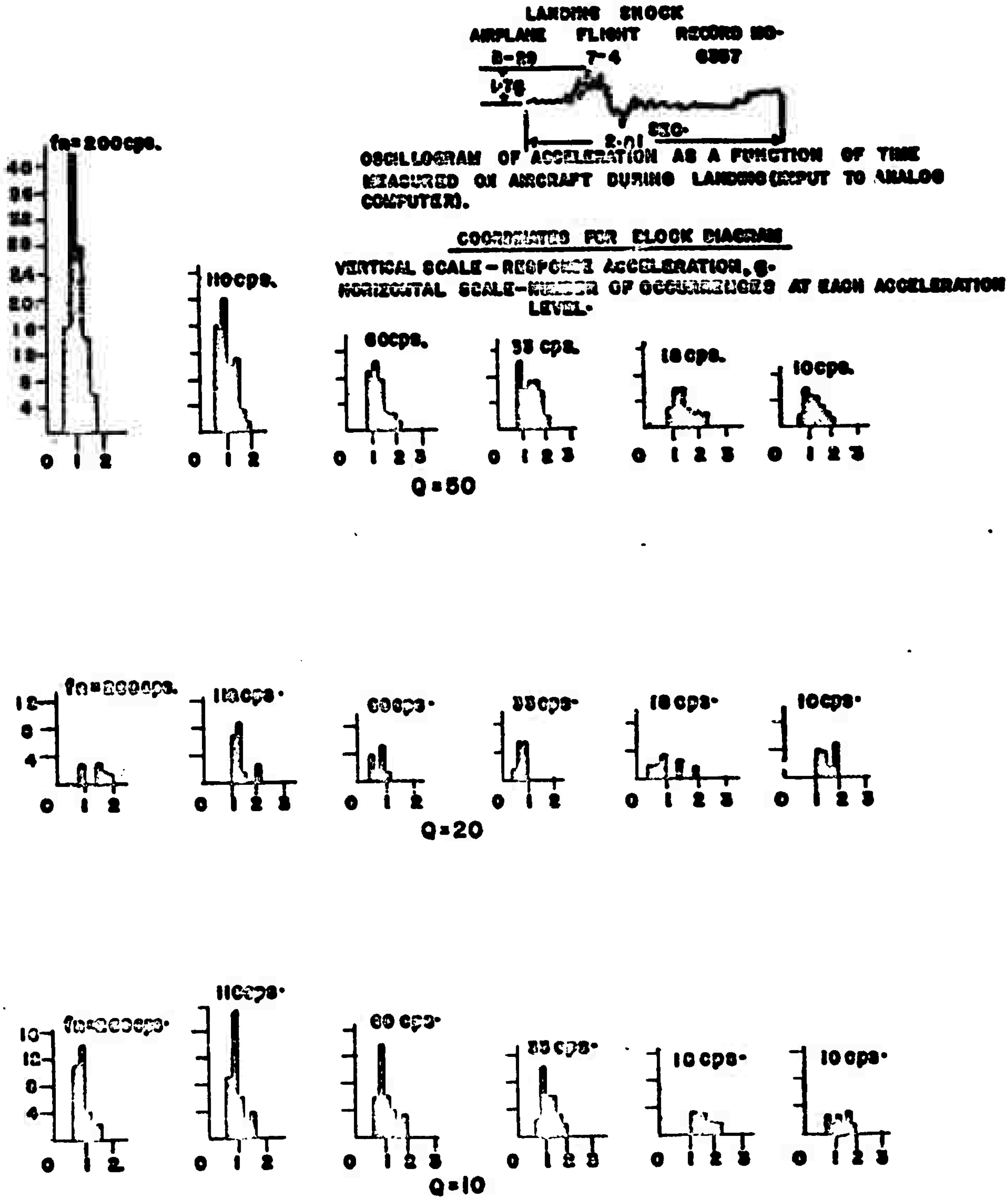


FIGURE III-2. BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF OCCURRENCES FOR SYSTEMS HAVING Q = 10, 20, 50, SUBJECTED TO B-29 LANDING SHOCK.

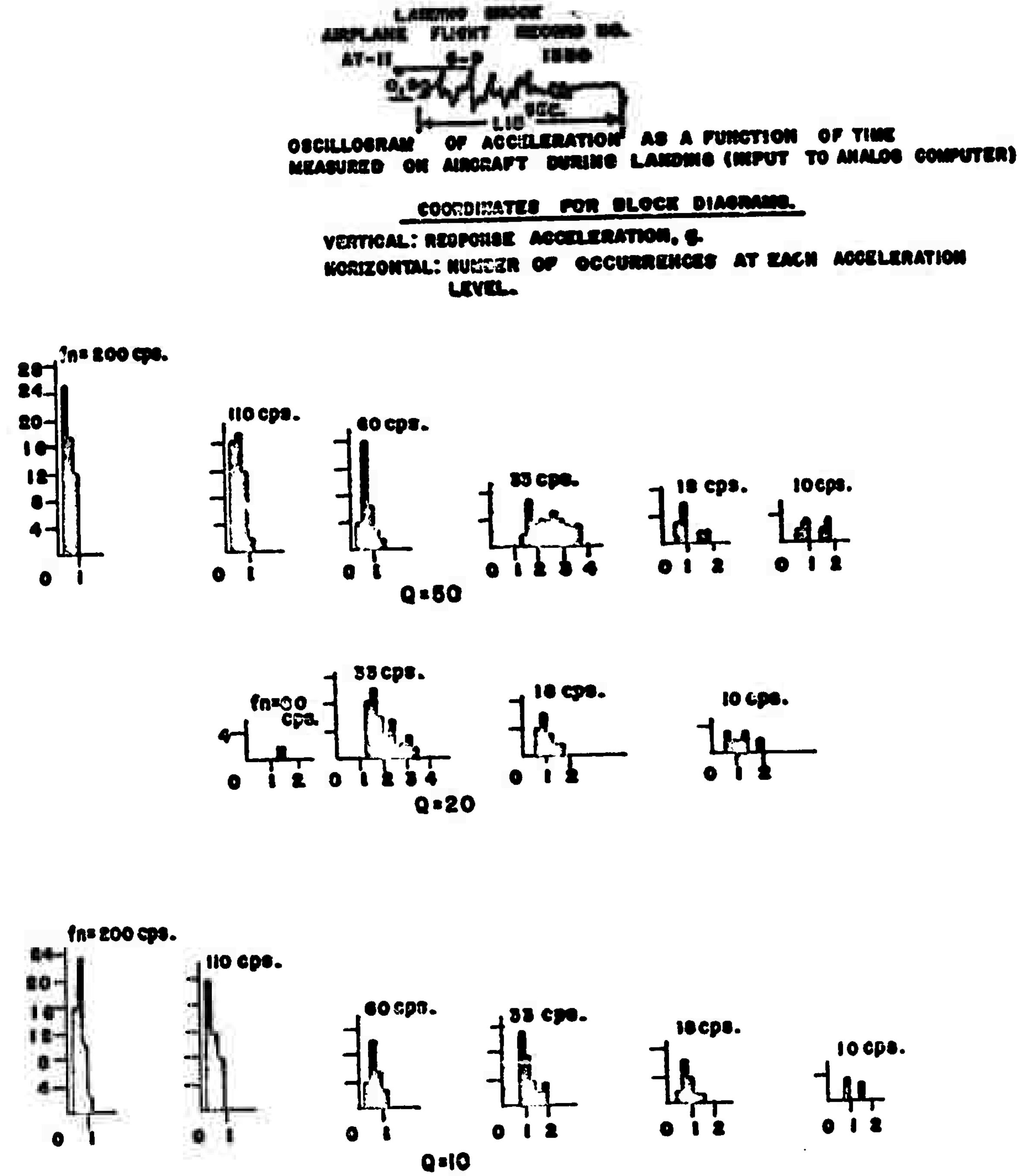


FIGURE I-3. BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF NUMBER OF OCCURRENCES FOR SYSTEMS HAVING Q=10,20,Q 50, SUBJECTED TO AT-11 LANDING SHOCK.

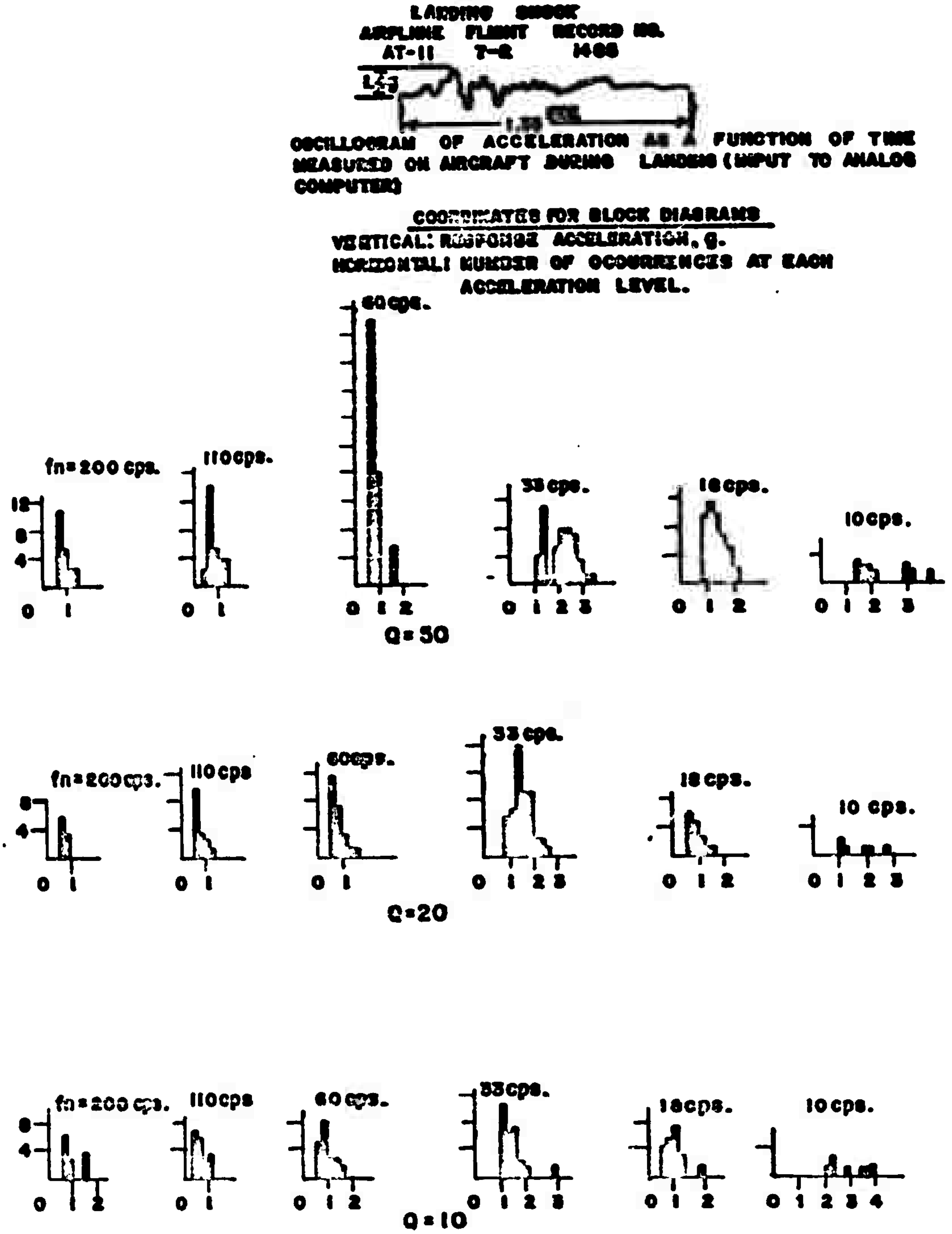


FIGURE I-4· BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF NUMBER OF OCCURRENCES FOR SYSTEMS HAVING Q=10,20, 50, SUBJECTED TO AT-11 LANDING SHOCK.

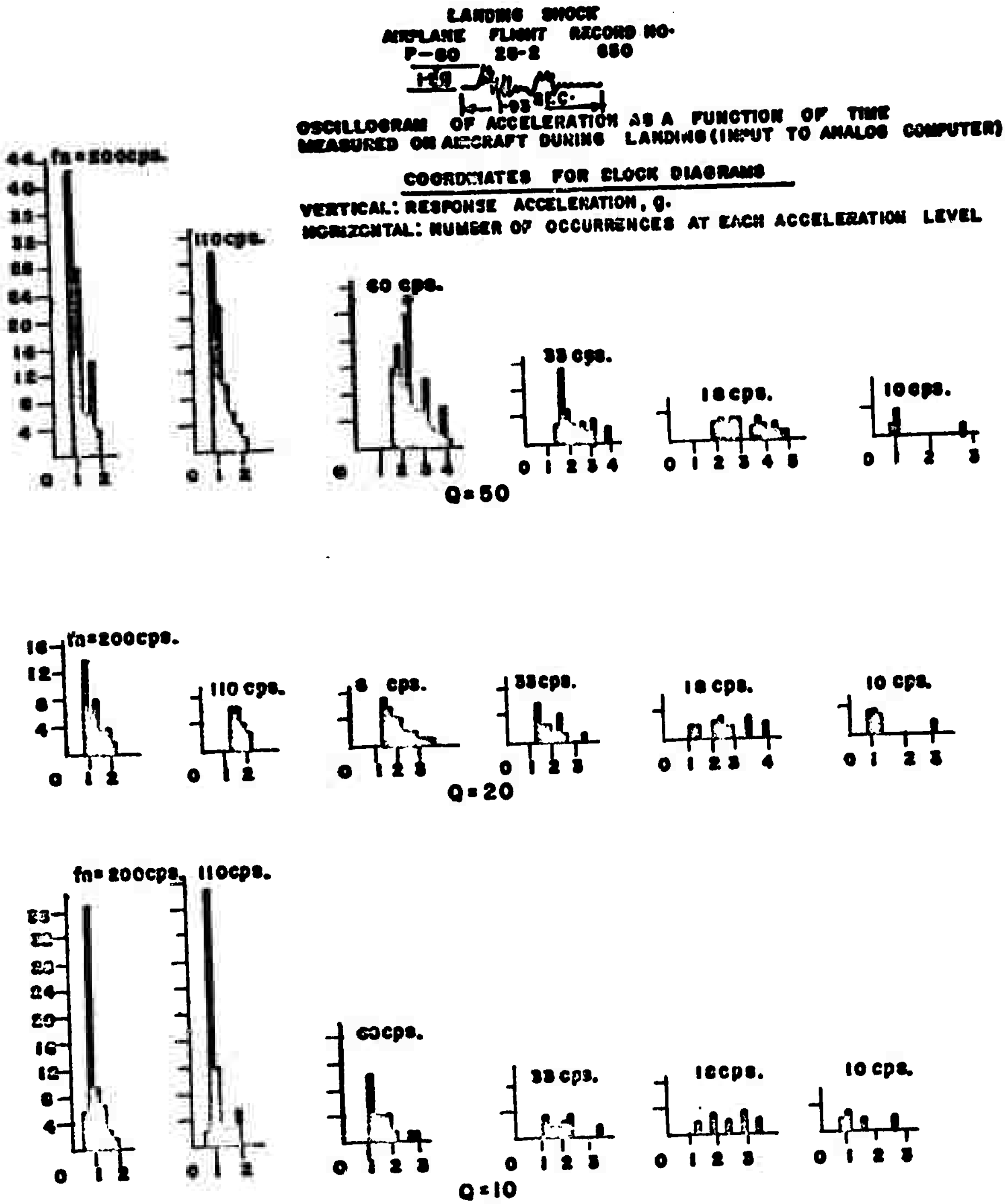


FIGURE I-5. BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF NUMBER OF OCCURRENCES FOR SYSTEMS HAVING Q = 10, 20, & 50, SUBJECTED TO P-80 LANDING SHOCK.

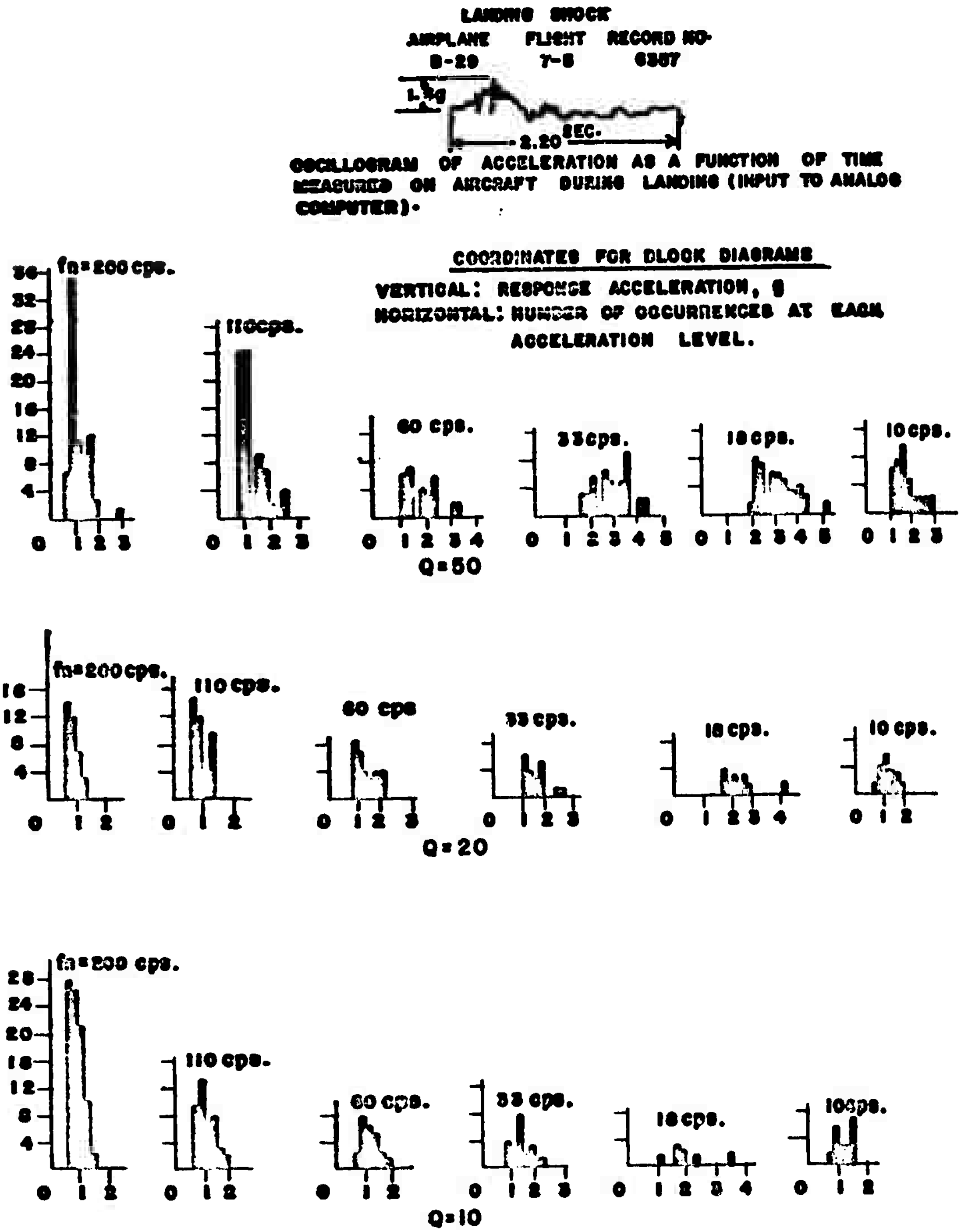


FIGURE I-6. BLOCK DIAGRAMS OF RESPONSE ACCELERATION AS A FUNCTION OF NUMBER OF OCCURRENCES FOR SYSTEMS HAVING Q=10, 20, 50, SUBJECTED TO B-29 LANDING SHOCK.

APPENDIX II

ANALOG COMPUTER

Where transient vibration or shock is included in the environment, a definition of the environment may be obtained by recording the time history of displacement, velocity, or acceleration. In the analysis carried out in this report, the environment is defined in terms of acceleration as shown by the acceleration-time curves set forth as insets to the Figures of Appendix I. As explained in Section 9 of this Report, the severity of the environment is evaluated by determining the acceleration response of single-degree-of-freedom systems having various natural frequencies and various degrees of damping. This Appendix describes the analog methods used to obtain the responses.

The system being investigated is shown by the schematic diagram in Figure II-1 where x indicates the displacement of the airframe structure and y indicates the displacement of any equipment component attached to the airframe. The mass m, stiffness k, and damping coefficient c define the physical characteristics of the component whose response is to be determined. The differential equation of motion of the mass m is written as follows:

$$m\ddot{y} = c(\dot{x}-\dot{y}) + k(x-y) \qquad \text{(II-1)}$$

Letting the relative motion of y with respect to x be represented by a new variable $\delta = x - y$, equation II-1 may be rewritten as follows:

$$m(\ddot{x}-\ddot{\delta}) = c\dot{\delta} + k\delta \qquad \text{(II-2)}$$

This equation may be written:

$$m\ddot{y} = c\dot{\delta} + k\delta \qquad \text{(II-3)}$$

Collecting all terms containing δ in equation II-2 on the right hand side of the equation:

$$m\ddot{x} = m\ddot{\delta} + c\dot{\delta} + k\delta \qquad \text{(II-4)}$$

Equation (II-4) may be expressed in block diagram form as shown in Figure II-2. In this Figure, the blocks indicated by J perform the operation of integration, those indicated

by C perform the operation of multiplication by a constant wherein the constant is given in parentheses below the block, and those indicated by A perform the operation of addition of several quantities. All operations take place in the direction indicated by the arrows. The environment is defined by $\ddot{X}$ which is, in effect, multiplied by the coefficient m to obtain the input m $\ddot{X}$ to the analog computer. The output is m $\ddot{Y}$, in accordance with equation (II-3). This is converted to the desired response acceleration $\ddot{Y}$ by dividing, in effect, by the constant m used initially to convert $\ddot{X}$ to m $\ddot{X}$.

The electrical analogy method consists of employing electrical quantities corresponding to the variable terms in equation (II-4), and performing the indicated integration, multiplication, or addition of these electrical quantities in the order indicated by the block diagram of Figure II-2. Equipment for performing these operations electrically is available commercially from several sources. The project described in this report does not justify the cost of analog computer components of relatively great accuracy, because the acceleration records which define the environments in aircraft are generally not precise. Taking these limitations into consideration, an analog computer suitable for this investigation has been assembled from components manufactured by George A. Philbrick Researches, Inc. of Boston. Each component represented by one of the blocks in Figure II-2 is self-contained; it embodies a housing and convenient output and input terminals together with a dial for adjusting the characteristics of the component. These components, together with the necessary power supply, rack and attaching cables, are assembled together as shown at the right hand side of the photograph constituting Figure II-3.

The input to the computer is a voltage proportional to $\ddot{X}$ which varies with time in an irregular manner. It was necessary to construct a special function generator to generate a voltage proportional to the ordinate on the acceleration-time diagrams shown as insets to the Figures in Appendix I. A special function generator suitable for this purpose was designed and constructed by Professor James R. Reswick of the Mechanical Engineering Department of the Massachusetts Institute of Technology. This function generator is shown with its cover elevated on the wheeled table at the center of the photograph in Figure II-3.

A schematic view of the function generator is shown in Figure II-4. It is comprised basically of an oscilloscope, a transparent drum mounted on a turn table, and a photo multiplier tube. The record which is being studied must be traced with a heavy dark line on transparent material and then attached to the transparent drum. The record is rotated in front of the oscilloscope screen, and the photo multiplier tube causes the beam of the cathode ray tube to follow the dark line of the record as it is rotated in front

of the tube. This occurs because the beam has a constant upward potential; when it stays above the dark line, the photo multiplier tube feeds a voltage back into the vertical DC amplifier of the oscilloscope and drives the beam down behind the dark line. Consequently, the beam of the oscilloscope rides below the edge of the dark line of the record, and the varying voltage necessary to hold the beam on the dark line can be picked off the plate terminals at the back of the oscilloscope.

The circumference of the transparent drum is approximately 36 inches. The function generator thus accommodates any random record of this length and converts it into a steady-state function because it repeats itself at each revolution of the drum. The voltage which is picked off the plate terminals of the oscilloscope thus represents this steady-state function; it can be viewed on another oscilloscope or used as the input to the analog computer. In the work reported herein, a DuMont Type 322 Dual Beam Oscillograph is used with the function generator and analog computer so that both the input to and the output from the analog computer can be observed simultaneously.

An investigation was carried out to establish the equivalence of the response of a system as determined by the function generator and analog computer to the actual response of a mechanical structure subjected to a shock motion. This was done to gain confidence in the accuracy and validity of the analog computer and function generator. The mechanical structures used for this investigation were five cantilever beams, ranging in natural frequency from 30 to 500 cps. The beams were subjected to the shock motion produced by the Type 150 VD Shock Machine, and the strain in each beam was measured by a SR-4 strain gage feeding through a carrier type amplifier and detector into the channel of a DuMont Type 322 Dual Beam Oscillograph. The outputs of the strain gages were recorded photographically. Simultaneously with the measurement of strain in each cantilever beam, the time history of the acceleration on the elevator of the shock machine were recorded by a Calidyne accelerometer having a natural frequency of 1050 cps placed adjacent to the beam.

A typical record obtained from the tests of the cantilever beams is shown in Figure II-5(a). The 60 cps calibration traces at the upper part of the record each have an amplitude of 30g peak-to-peak. The next trace is the accelerometer output and the bottom most trace is the strain of a cantilever beam having a natural frequency of 493 cps. The accelerometer record was enlarged four times by photographic means, traced on acetate, and attached to the transparent drum of the function generator. The analog computer was set to compute the response of a system having a natural

frequency of 493 cps to the accelerometer record as reproduced by the function generator. Figure II-5(b) shows the results of this work. The response as determined by the analog computer is shown as the upper trace in Figure II-5(b), and the reproduction of the accelerometer record by the function generator is shown as the lower trace. Comparison of Figures II-5(a) and II-5(b) shows excellant correlation between the experimentally obtained response of the beam system and the results obtained from the analog computer. The slight wavering of the lower trace in Figure II-5(b) is attributed to a bad contact in the time generating potentiometer of the function generator. This has since been corrected.

It was noted that systems with small degrees of damping could not be studied accurately on the analog computer because oscillations were present from the solution generated at the immediately preceding revolution of the drum of the function generator. This introduced a slight error because these oscillations were superimposed upon the solution generated at each subsequent revolution of the drum. The function generator has been reworked to incorporate a "clamping device" which automatically zeros the computer after each complete revolution of the record drum.

The analog computer described in this Appendix has received extensive use in the investigation and evaluation of environmental conditions defined by transient records. Approximately 250 response accelerations have been determined, representing the responses of systems having various natural frequencies and damping constants to the landing shocks recorded in various aircraft. The analog computer and the accompanying function generator have been found reliable and convenient to operate. For the purposes of this study, the analog computer is considered to be practical in evaluating transient conditions.

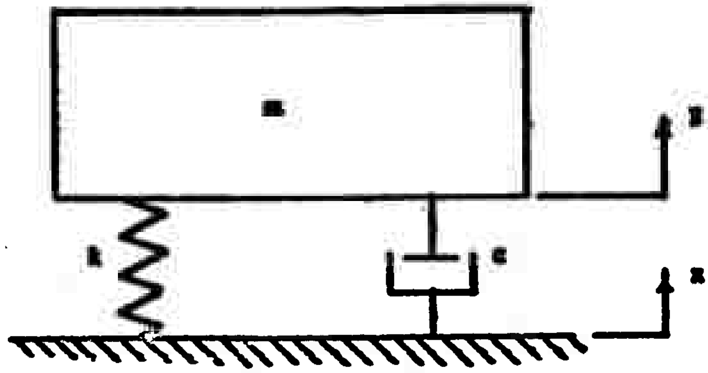


FIGURE II-1. DAMPED, SINGLE-DEGREE-OF-FREEDOM SYSTEM WHOSE RESPONSE IS DETERMINED ON ANALOG COMPUTER.

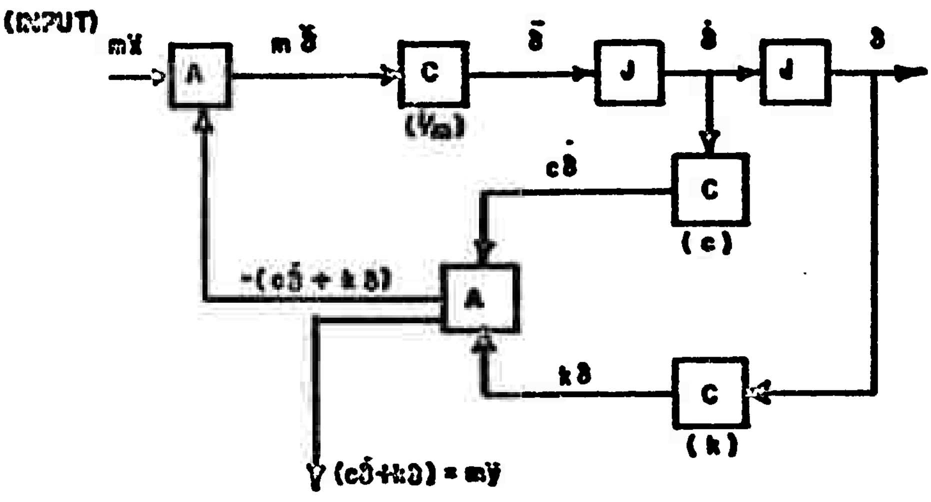


FIGURE II- 2. OPERATIONAL BLOCK DIAGRAM FOR EQUATION (II-4).

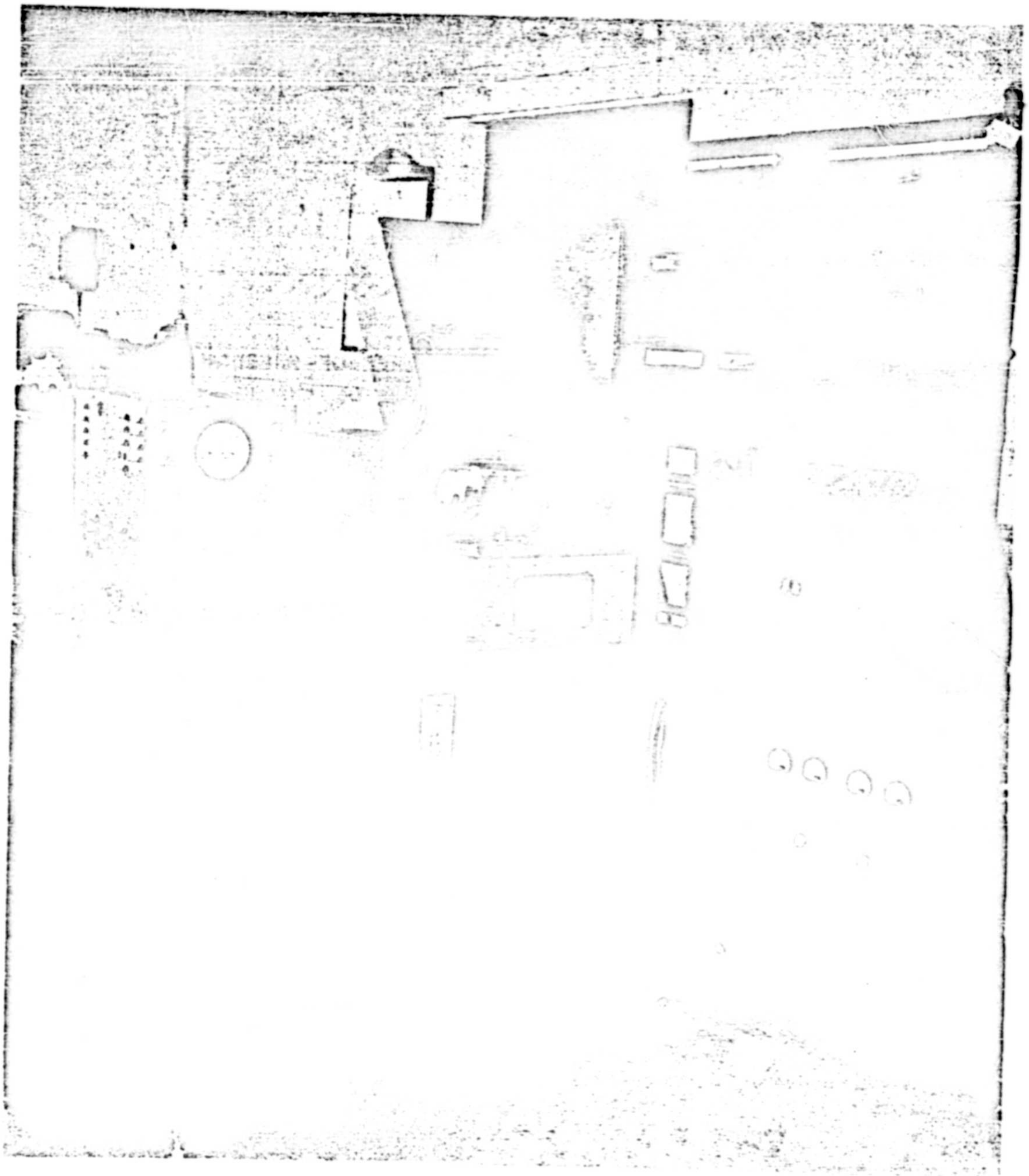


Figure II-3. Photograph showing analog computer and function generator.

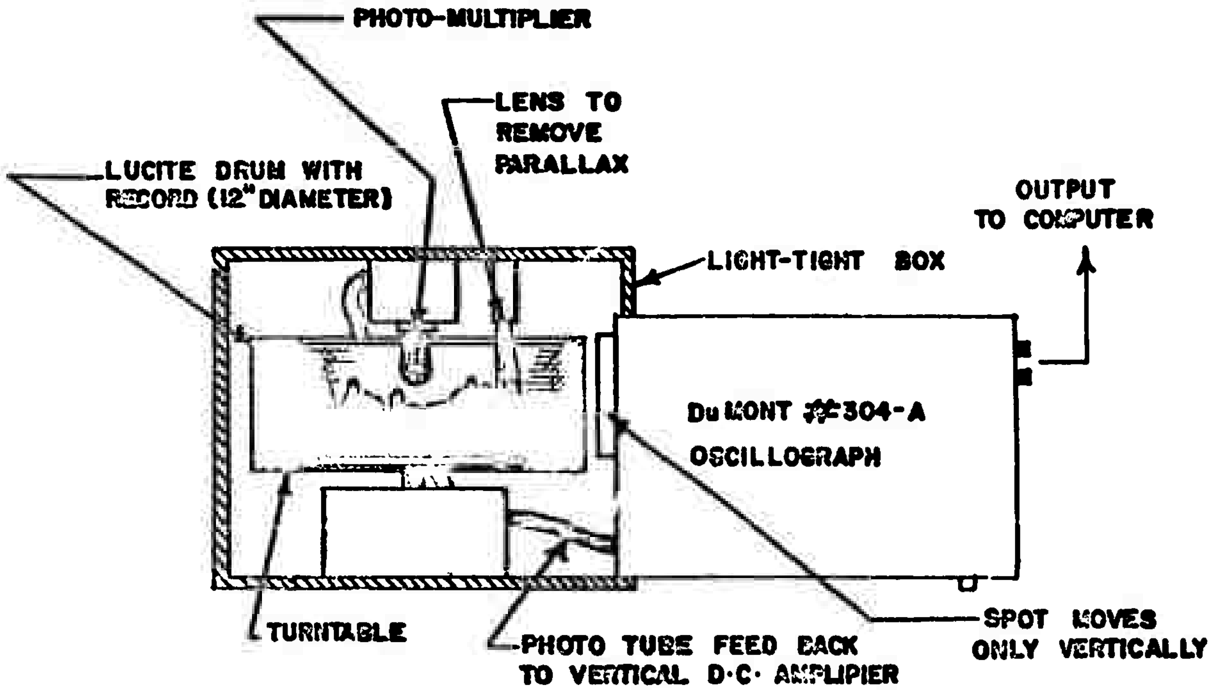
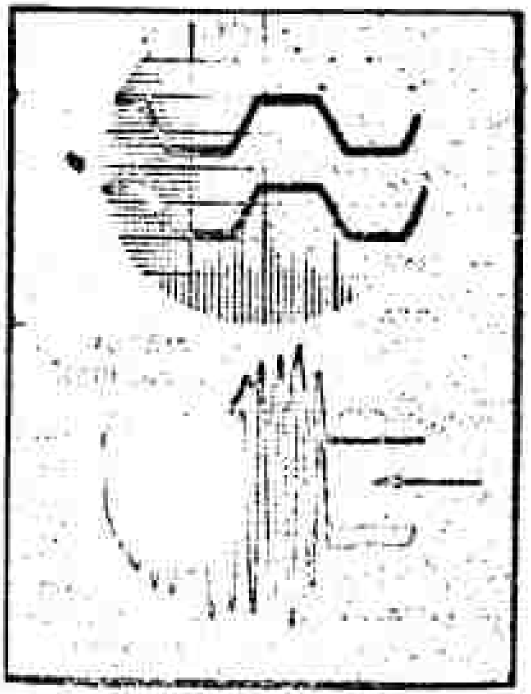
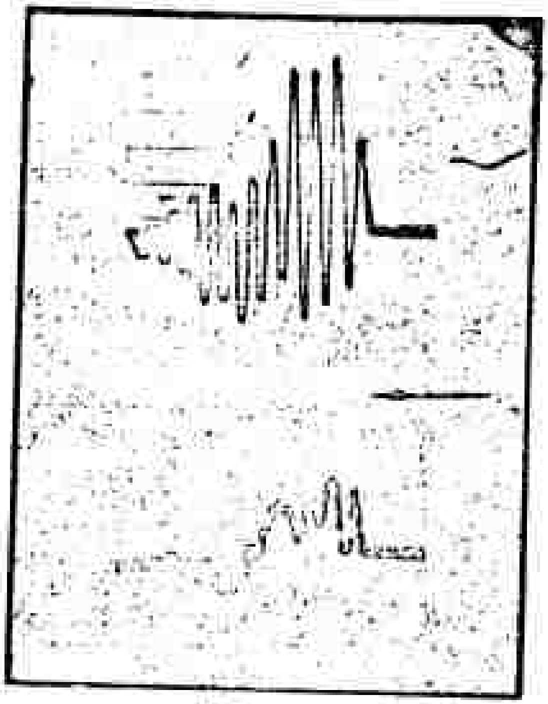


FIGURE II-4. SCHEMATIC VIEW OF FUNCTION GENERATOR



(a)

(1) Calibration trace - 60 cps. and 30g peak-to-peak.
(2) Same as (1).
(3) Acceleration measured on Table of Type 150-400 VD Shock Machine.
(4) Strain in cantilever beam mounted on Table - Natural frequency = 493 cps.



(b)

(1) Response of analog circuit with natural frequency of 493 cps. to trace (2).
(2) Trace (a) 3 as reproduced by function generator

Figure II-5. Comparison of response of analog circuits to actual physical response.

APPENDIX III

BIBLIOGRAPHY

Books

1. Battelle Memorial Institute Staff, "Prevention of Fatigue of Metals", John Wiley and Sons, Inc., New York. (1941)

2. Crede, C. E., "Vibration and Shock Isolation", John Wiley and Sons, Inc., New York, 1951, p. 33.

3. Fry, T. C., "Probability and Its Engineering Uses", D. Van Nostrand Co., New York, 1928.

4. Manley, R. G., "Fundamentals of Vibration Study", John Wiley and Sons, Inc., New York, 1943.

5. "Mechanical Properties of Metals and Alloys", National Bureau of Standards, Circular C-447, December 1943.

6. Murray, W. M., "Fatigue and Fracture of Metals - A Symposium", Technology Press of M.I.T. and John Wiley and Sons, Inc., New York. (1952)

Periodicals

7. Bennett, G. S., "On Multiple Excitation of an Elastic System", J. Acoustical Society of America, Vol. 23, No. 2, March 1951.

8. Bennett, R. R. & McCann, G. D., Jr., "Vibration of Multifrequency Systems During Acceleration Through Critical Speeds", ASME Paper 49-APM-10.

9. Biot, M. A., "Analytical & Experimental Methods in Engineering Seismology", Trans. ASCE, Vol. 108, 1943, p.365.

10. Biot, M. A., "A Mechanical Analyzer for the Prediction of Earthquake Stresses", Bulletin of the Seismological Society of America. (1940)

11. Bisplinghoff, R. L., Pian, T. H. H., & Levy, L. I., "A Mechanical Analyzer for Computing Transient Stresses in Airplane Structures", J. App. Mech., Vol. 17, No. 3., Sept. 1950, p.310.

12. Cochardt, A. W., "A Method for Determining the Internal Damping of Machine Members", ASME Paper No. 53-A-44.

13. Conrad, R. W. & Vigness, I., "Calibration of Accelerometers by Impact Techniques", Inst. Soc. Of Am., Paper No. 53-11-3.

14. DenHartog, J. P., "The Use of Models in Vibration Research", Trans. ASME Paper No. APM-54-14.

15. Evaldson, R. L., Ayre, R. S., Jacobsen, L. S., "Response of an Elastically Non-Linear System to Transient Disturbances", J. Frank. Inst., Vol. 248, No. 6, Dec. 1949.

16. Frankland, J. M., "Effects of Impact on Simple Elastic Structures", Proc. SESA, Vol. VI, No. II.

17. Hsia, Pei-Su, "A Graphical Analysis for Non-linear Systems", Proc. Inst. E. E., Vol. 99 Part II, No. 68, April, 1952.

18. Housner, G. W., "Characteristics of Strong-Motion Earthquakes", Bul. Seis. Soc. of Am., Vol. 37, No. 1, Jan. 1947.

19. Lazan, B. J., "Effect of Damping Constants and Stress Distribution on the Resonance Response of Members", ASME Paper No. 52-2-8.

20. Lazan, B. J., "Some Mechanical Properties of Plastics and Metals Under Sustained Vibrations", Trans. ASME, February, 1943.

21. Levy, S., & Kroll, W. D., "Response of Accelerometers to Transient Accelerations", Journal of Research, National Bureau of Standards, Vol. 45, No. 4, October 1950. (Research Paper No. 2138).

22. Lewis, F. M., "Vibration During Acceleration Through a Critical Speed", Trans. ASME, 1932, p.253-61.

23. McCann, G. D. & MacNeal, R. H., "Beam-Vibration Analysis with the Electric Analog Computer", ASME Paper No. 49-SA-3.

24. Meuser, R. R. & Weibel, E. E., "Vibration of a Non-linear System During Acceleration Through Resonance", ASME, Paper No. 47-A-53.

25. Mindlin, R. D., Stubner, F. W., & Cooper, H. L., "Response of Damped Elastic Systems to Transient Disturbances", Proc. Soc. Exp. Stress Anal., Vol. V, No. II. (1947)

26. Miner, M. A., "Cumulative Damage in Fatigue", Jour. App. Mech., Vol. 13, No. 3, Sept. 1945, p. A-159.

27. Pardue, T. E., Melchor, J. L., and Good, W. B., "Energy Losses and Fracture of Some Metals Resulting from a Small Number of Cycles of Strain", Proc. SESA, Vol. VII, No. II.

28. Ramberg, W. & McPherson, A. E., "Experimental Verification of Theory of Landing Impact", BU. Stds. Paper RP 1936, Vol. 41, Nov. 1948.

29. Richart, F. E., Jr., and Newmark, N. M., "An Hypothesis for the Determination of Cumulative Damage in Fatigue", Proc. ASTM, Vol. 48, 1948.

30. Roberson, R. E., "Vibrations of a Clamped Circular Plate Carrying Concentrated Mass", J. App. Mech., Dec. 1951.

31. Scanlan, R. H., "An Analytical Study of the Landing Shock Effect on an Elastic Airplane", Jour. Aero. Sci., Vol. 15, No. 5, May 1948.

32. Shapiro, H. and Hudson, D. E., "The Measurement of Acceleration Pulses with the Multifrequency Reed Gage", ASME Paper 53-SA-8.

33. Shou-Ngo.Tu, "Dynamic Loads in Airplane Structures During Landing", J. Aero Sci., VI 3 n7, July 1946, p. 381.

34. Sinclair, G. M., and Dolan T. J. "Effect of Stress Amplitude of Statistical Variability in Fatigue Life of 75S-T6 Aluminum Alloy", ASME Paper No. 52-A-82.

35. Smith, A. D. N., "An Electronic Instrument for the Measurement of the Damping Capacity of Materials", J. Sci. Instr., Vol. 28, 1951.

36. "Some Simple Electrical Analogies", Product Engineering, July 1947, pp. 126-130.

37. Soroka, W. W., "Experimental Aids in Engineering Design Analysis", Mech. Eng., Vol. 71, No. 10, Oct. 1949.

38. Soroka, W. W., "Mechanical and Electrical Analogues For Vibration Isolation Problems", Prod. Eng., Feb. 1950.

39. Starkey, W. L., & Marco, S. M., "A Concept of Fatigue Damage", ASME Paper No. 53-A-143.

40. Weibull, W., "Statistical Design of Fatigue Experiments", J. App. Mech., March 1952, Vol. 19, No. 1, P. 109.

41. Wilbur, J. B., "The Action of Impulsive Loads on Elastic Structures", Boston Society of Civil Engineers, July 1946.

42. Woodson, J. B., "The Dynamic Response of a Simple Elastic System to Antisymmetric Forcing Function Characteristic of Airplanes in Unsymmetric Landing Impact", ASME Paper No. 48-A-16.

Documents

43. "Aeronautical Structures Laboratory Report on Repeat-Load Tests of Model AD Wing Outer Panels, Naval Air Experiemtnal Station, Naval Air Material Center, Philadelphia, Report No. ASL NAM DE-247, January 6, 1953.

44. "Aircraft Radio Equipment Airworthiness", Civil Aeronautics Manual 16, Department of Commerce Civil Aeronautics Administration, Washington, February 13, 1941.

45. Alford, J. L., Housner, G. W., and Martel, R. R., "Spectrum Analyses of Strong-Motion Earthquakes", Earthquake Research Laboratory, California Inst. of Tech, First Tech. Report under Contract N6onr-244, August 1951.

46. Carnahan, K. R., "Interim Engineering Report on Investigation of Vibration and Shock Requirements for Airborne Electronic Equipment", Curtiss-Wright Corp. Report No. 120X-4, May 27, 1950, AF 33(038)-7379.

47. "A collection of Considerations Towards a Rational Mechanical Engineering Practice for Communication Equipment", Signal Corps Engineering Laboratories, Bradley Beach, New Jersey, October 3, 1947.

48. "Cooperative Freight Truck and Snubber Research Program and Related Freight Car Research Programs", Summary Report of Road Tests, Association of American Railroads, Operations and Maintenance Department, Mechanical Division, Report No. f3800, June 30, 1951.

49. Crim, A. D., and Hazen, M. E., "Normal Accelerations and Operating Conditions Encountered by a Helicopter in Air-Mail Operations", NACA Technical Note 2714, June 1952.

50. Cross, W. H. and McWhirter, M., "Railroad Switching Shock", Association of American Railroads, Operations and Maintenance Department.

51. Davidson, S. and Adams, E. J., "A Theoretical Study of the Multifrequency Reed Gage for Measuring Shock Motion", The David Taylor Model Basin, U. S. Navy Report 613, July 1949.

52. "The Destructive Effects of Vibrations and Shocks", Navy Department, Bureau of Ships, NavShips 900,036, April 1944.

53. "Differential Analyzer", Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey, February 28, 1952, Information Bulletin No. 1.

54. "Digital and Analog Computers and Computing Methods", Symposium at the 18th Applied Mechanics Division Conference of the ASME held at The University of Minnesota, June 18-20, 1953.

55. "Final Engineering Report on Investigation of Vibration and Shock Requirements for Airborne Electronic Equipment", North American Aviation, Inc., Report No. 120X-7, March 30, 1951, Contract No. AF 33(038)-7379.

56. Firmage, D. A., "Transportation Shock and Vibration Studies", Engineering and Industrial Experiment Station, University of Florida, Final Report under Contract No. DA 44-009 eng-460, February 25, 1952.

57. Fischer, E. G., "Preliminary Report on the Mathematical Analysis of the Shock Problem," Research Report No. SR-136 from Westinghouse Research Laboratories.

58. Floor, Ir. W. K. G., "Reliability of the Drop Weight Reduction Method for Simulating Wing Lift Effect in Landing Gear Drop Tests," National Aeronautical Research Institute, Amsterdam, Rpt. S.340, July 30, 1948.

59. Gee, S. M., Guttwein, G. K., and Priebe, F. K., "Evaluation of the Behavior of Electronic Equipment During Vehicular Transportation", Engineering Report No. E-1098, Signal Corps Engineering Laboratories, Fort, Monmouth, New Jersey. August 25, 1952.

60. Gee, S. M., Guttwein, G. K., and Priebe, F. K., "Investigation of Shock and Vibration Encountered During Railroad Switching Operation", Engineering Report No. E-1093, Signal Corps Engineering Laboratories Fort Monmouth, New Jersey, June 3, 1952.

61. Geiger, P. H., "Vibration Damping", University of Michigan, Office of Naval Research, Contract N6 onr-23211, October 1, 1950.

62. Goldberg, B. and Pardue, T. E., "Resistance of Materials to Mechanical Shock", Naval Research Laboratory, Report 3828, July 13, 1951.

63. Hendrickson, A. C., " Slopeline-Graphical Solutions of Ordinary Non-Linear Differential Equations with Engineering Applications", M.I.T. Thesis, E. E. Dept., 1952.

64. Hoffer, E. S., "An Electronic Method of Approximating the Frequency Spectra of Transient Functions", Naval Research Laboratory, Washington, D. C., Report F-3406, January 27, 1949.

65. Howard, D. M., and Smith, F. C., "Fatigue and Static Tests of Flush-Riveted Joints", NACA, Technical Note 2709, June 1952.

66. Hudson, D. E., Alford, J. L., and Housner, G. W., "Response of a Structure to an Explosive- Generated Ground Shock", California Institute of Technology, Earthquake Research Laboratory, A Report on research conducted under contract with the Office of Naval Research Contract N6 onr-244, September 1952, Third Technical Report.

67. Hudson, G. E., "A Method of Estimating Equivalent Static Loads in Simple Elastic Structures", David W. Taylor Model Basin, U. S. Navy Report 507, June 1943.

68. "Interim Engineering Report of Investigation of Vibration and Shock Requirements for Airborne Electronic Equipment", North American Aviation, Inc., Report No. 120X-6, January 7, 1951, Contract AF 33(038)-7379.

69. Jacobson, J. M., and Nietsch, H. E., "An Approach to the Analysis of Landing Test Data for Use in Fatigue Calculations", Glenn L. Martin Company, Presented at SESA Detroit, Michigan, May 19, 1949.

70. Kennard, D. C., Jr., "Measured Aircraft Vibration as a Guide to Laboratory Testing", AF Technical Report No. 6429, May 1951.

71. Kennard, D. C., Jr., and McIntosh, V. C., "Theory and Practice in Measuring Resonant Frequencies of Aircraft Generators and Alternators", USAF Technical Report No. 6125, August 1950.

72. Laverne, M. E., and Boksenbom, A. S., "Frequency Response of Linear Systems From Transient Data", National Advisory Committee for Aeronautics, Report 977, 1950.

73. Lazan, B., Gannett, A., Kirmser, P., Klumpp, J., and Brown, J., "Dynamic Testing of Materials and Structures with a New Resonance-Vibration Exciter and Controller", Wright Field, Technical Report 52-252, December 1952.

74. Lee, Norman E., "Mechanical Engineering as Applied to Military Electronic Equipment", Technical Memorandum No. M-1177, Signal Corps Engineering Laboratories, Fort Monmouth, New Jersey, June 22, 1949.

75. Levenson, M., and Sussholz, B., "The Response of a System With a Single Degree of Freedom to a Blast Load", The David W. Taylor Model Basin, U. S. Navy Report 572, December 1947.

76. "Lockheed P-80A Airplane - Flight Vibration Characteristics of Jettisonable, Wing Tip Fuel Tanks", Army Air Forces Air Technical Service Command, Wright Field, Memorandum Report No. MCREXA5-524-5, April 16, 1948.

77. McGolddrick, R. T., "Shock and Vibration Instrumentation for Ships", Symposium on Shock & Vibration Instrumentation at the 17th Applied Mechanics Division Conference held at Penn. State College, State College, Pa., June 19-21, 1952.

78. Muller, J. T., "Transients in Mechanical Systems", American Telephone and Telegraph Company, New York.

79. Powell, H. R., "Drop-Table Type Shock Tester", Boston University Upper Atmosphere Research Laboratory, Technical Note No. 9, August 15, 1950.

80. "Prediction of Electronic Failure", National Bureau of Standards Technical News Bulletin, Vol. 36, No. 12, December 1952.

81. Press, H., "An Approach to the Prediction of the Frequency Distribution of Gust Loads on Airplanes in Normal Operations", NACA Technical Note 2660, April 1952.

82. Press, H. and McDougal, Robert L., "The Gust and Gust-Load Experience of a Twin-Engine Low-Altitude Transport Airplane in Operation on a Northern Transcontinental Route", NACA Technical Note 2663. April 1952.

83. "Repeat-Load Tests of Model F6F Airplane Wings", Aeronautical Structures Laboratory, Naval Air Experimental Station, Naval Air Material Center Philadelphia, Report No. ASL NAM DE-281, Nov. 20, 1952.

84. "Republic F-84 Airplane Vibration Measurements on General Electric J-35 Engine Installations", Aircraft Laboratory, Wright Field, Memorandum No. MCREXA5-4554-2-1, October 1, 1948.

85. "Resilient Mountings for Reciprocating and Rotating Machinery", Illinois Institute of Technology, Engineering Report No. 2, Cont. N7-onr-32904, June 14, 1950.

86. "Response of a Yielding Vibratory System of Transient Forcing Functions (I)", Vibration Research Laboratory, School of Engineering, Stanford University, Structural Dynamics Technical Report No. 11, Navy Contract N6-ORl 154, Task Order 1.

87. "Response of a Yielding Vibratory System to Transient Forcing Functions (II)", Vibration Research Laboratory, School of Engineering, Stanford University, Structural Dynamics Technical Report No. 14, Navy Contract N6-ORl 153, Task Order 1.

88. Sinclair, G. M., and Dolan, T. J., "Use of a Recrystallization Method to Study the Nature of Damage in Fatigue of Metals", Research Project of the Department of Theoretical and Applied Mechanics, Engineering Experiment Station, University of Illinois, Technical Report No. 21, February 1951.

89. Steiner, R., and Persh, D. A., "Normal Acceleration and Associated Operating Conditions on Four Types of Commercial Transport Airplanes from VGH Data Available as of September 1951", NACA Research Memorandum L52A28, May 19, 1952.

90. Trotter, W. D., "Flight Vibration and Acceleration Measurements", Prepared by Boeing Airplane Company, January 29, 1952.

91. Trotter, W. D., Kute, C. H., and Merritt, J. E., "Structural and Equipment Vibration Measurements", Prepared by Boeing Airplane Company, October 3, 1952.

92. "Vibration Characteristics Measured at Compass Transmitter Location in North American Type F-86-A5 Airplane", Wright Air Development Center, Technical Memorandum Report WCLE-52-7, January 16, 1953.

93. "Vibration Recording and Analysis Used in the Association of American Railroads Test Train, Clinton, Illinois", Signal Corps Engineering Laboratories, Technical Memorandum N. M-1321, October 20, 1950.

94. Vigness, I., "Measurements of Mechanical Shock by Peak-Reading Instruments," Naval Research Laboratory, Report No. 3818, May 25, 1951.

95. Wagenseil, W., "The destructive Effects of Vibration and Shocks," Bureau of Ships, NavShips 900,036, April, 1944.

96. Wagenseil, W., "Memorandum Concerning the Mounting of Electrical Equipment Used by Armed Forces on Shock Mounts", Bell Telephone Labs, August 21, 1943.

97. Walker, W. G., and Schumacher, P. W. J., "An Analysis of the Normal Accelerations and Airspeeds of a Two-Engine Type of Transport Airplane in Commercial Operations on Routes in the Central United States - From 1948 to 1950", NACA Tech. Note 2735, July 1952.

98. Walker, W. G., and Steiner, R., "Summary of Acceleration and Airspeed Data from Commercial Transport Airplanes During the Period from 1933 to 1945", NACA Technical Note 2625, February 1952.

99. Walsh, J. P., and Blake, R. E., "The Equivalent Static Accelerations of Shock Motions", Naval Research Laboratory, Report No. F-3302, June 21, 1948.

100. Weaver, P. R., and Hoeschele, D. F., "A Vibration Simulator for Bureau of Aeronautics for Department of the Navy", National Bureau of Standards Report No. 2582, June 1953.

101. Welch, W. P., "Impact Considerations in Design", Scientific Paper No. 1337 from Westinghouse Research Laboratories, November 17, 1947.

102. Welch, W. P., "Mechanical Shock on Naval Vessels", NavShips 250-660-26, Bureau of Ships, Washington, D. C., March 1, 1943.

103. Wilts, C. H. and McCann, G. D., "New Electric Analog Computers and Their Application to Aircraft Design Problems", California Institute of Technology.

104. Wrisley, D. L., and Knowles, W. S., "Investigation of Fasteners for Mounting Electronic Components", The Calidyne Company, Contract No. DA-36-039 SC-5545, April 15, 1951 - August 15, 1952, U. S. Army Signal Corps Engineering Laboratories, Fort Monmouth, N. J.

DISTRIBUTION LIST

Copies	Activities at Wright-Patterson Air Force Base
5	DSC-SA
2	WCOSI-4
1	WCOSI (RAND)
25	WCREO-2
1	WCRTO-5
2	WCLET (Attn: D. C. Kennard)
1	WCLNI
2	WCLNQ (Attn: C. W. McDowell)
3	WCLRI (Attn: M. J. McPeek)
1	WCLFA-3
2	WCSPE (Attn: R. F. Geist)
2	WCRTG (Attn: S. M. Birnbaum)
3	WCOSR (for possible foreign release)

Other Department of Defense Agencies

Air Force

Copies	
3	Commander AF Cambridge Research Center Attn: Documents Unit, CRQST-2 230 Albany Street Cambridge 39, Massachusetts
1	Commander Rome Air Development Center Attn: RCRES-4C Griffiss Air Force Base Rome, New York
1	Commander AF Missile Test Center Patrick Air Force Base Cocoa, Florida
1	Director Air University Library Maxwell Air Force Base, Alabama
1	AF Development Field Representative Naval Air Development Center Johnsville, Pennsylvania

Other Department of Defense Agencies (Cont'd)

Copies	
1	Chief Technical Library Office of Asst. Secretary of Defense (R&D) Room 3E 1065, The Pentagon Washington 25, D. C.
1	AF Development Field Representative Naval Research Laboratory Attn: Code 1010 Washington 25, D. C.
1	Chief European Office Air Research & Development Command Shell Building 60 Rue Ravenstein Brussels, Belgium

Army

Copies	
1	Commanding General Redstone Arsenal Attn: Technical Library-EC Huntsville, Alabama
1	Diamond Ordnance Fuse Laboratory Attn: J. D. Rosenberg Div. 52.3 Department of the Army Washington 25, D. C.
1	Commanding General Signal Corps Engineering Labs. Attn: Technical Documents Center Evans Signal Lab Area, Bldg 27 Belmar, New Jersey

DISTRIBUTION LIST (CONT'D)

Copies	Other Department of Defense Agencies
	Navy
1	Chief Bureau of Aeronautics Electronics Division Material Coordination Unit Navy Department Washington 25, D. C.
1	Chief Bureau of Aeronautics Attn: Technical Data Division TD-414 Navy Department Washington 25, D. C.
1	Chief Bureau of Aeronautics Attn: Code EL-4122, Mr. H. A. Colb Navy Department Washington 25, D. C.
1	Chief Bureau of Ships Code 816 Navy Department Washington 25, D. C.
1	U. S. Navy Electronics Laboratory Attn: Mr. A. H. Attebury Code 733 San Diego 52, California

Copies	Other U. S. [illegible] Organizations
1	National Bureau of [illegible] Div. 12.1 Miniaturization [illegible] Attn: Gustave Sha[illegible] Washington 25, D. C.
	Miscellaneous
1	Jet Propulsion [illegible] California Institute [illegible] Technology Attn: Dr. C. R. [illegible] 4800 Oak Grove [illegible] Pasadena 3, Califor[illegible]
5	[illegible] Corporation P. O. Box [illegible] 700 Pleasant Street Watertown 72, [illegible]